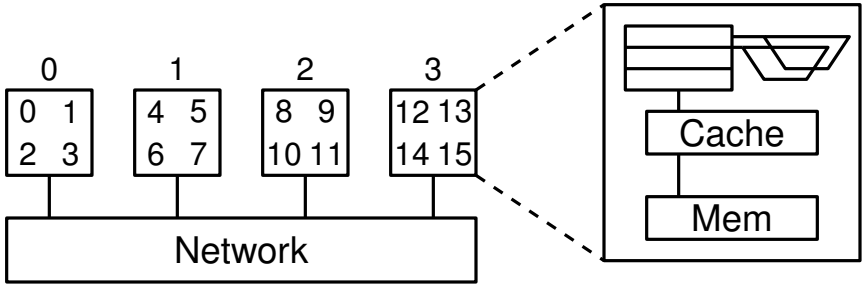
Karlsruhe Institute of Technology

Practical Massively Parallel Sorting

Michael Axtmann, Timo Bingmann, Peter Sanders, Christian Schulz

13th June 2015 @ SPAA 2015

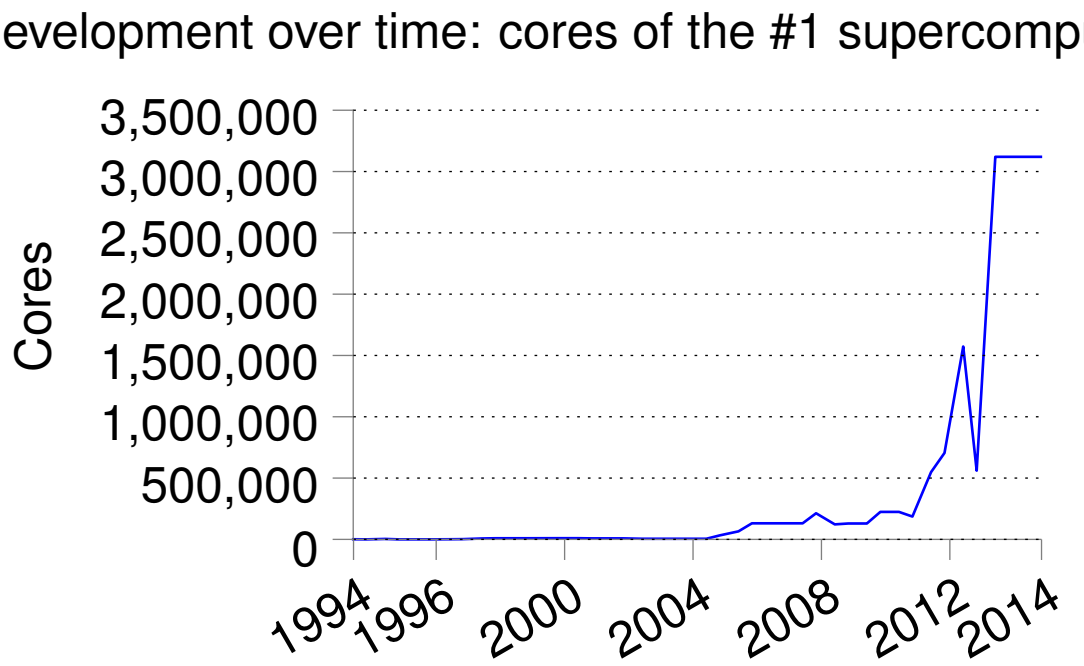
Institute of Theoretical Informatics – Algorithmics



Motivation

Example

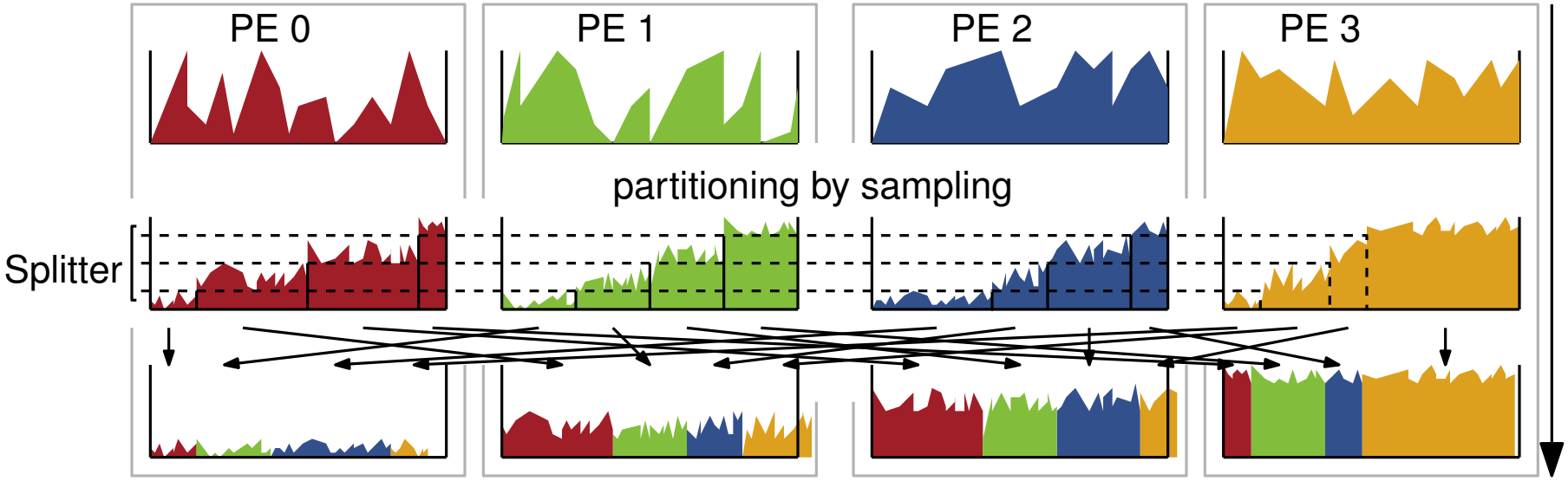
- Space-filling curves for load balancing in supercomputers
- Relatively small input



Data source: TOP500 November 2014

p-way Sample Sort

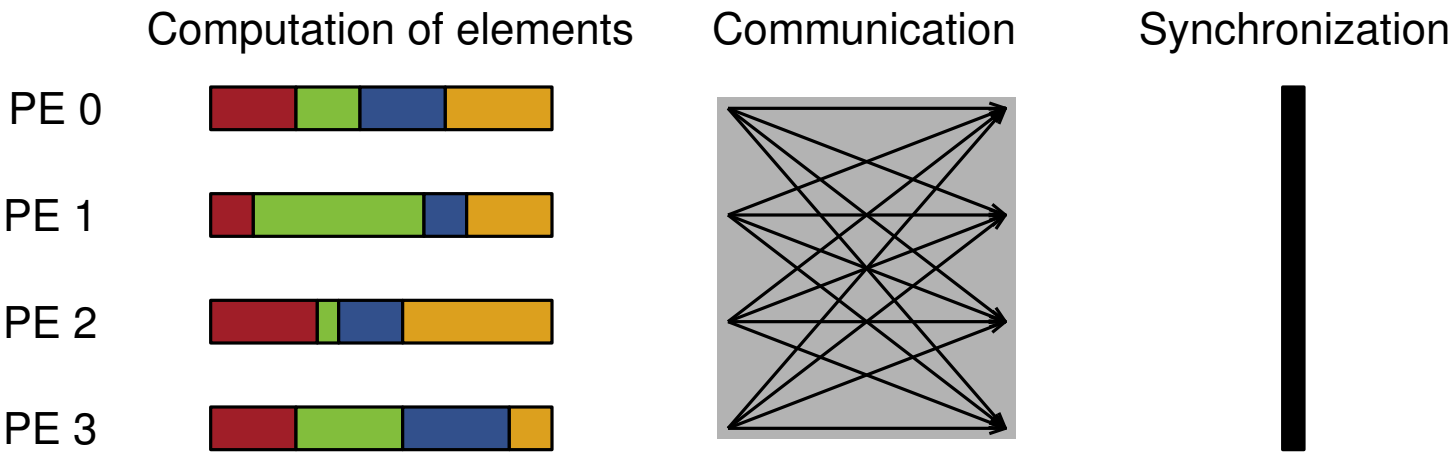
- Input: large *n*
- Many processing elements (PE) *p*
- Delivering data once



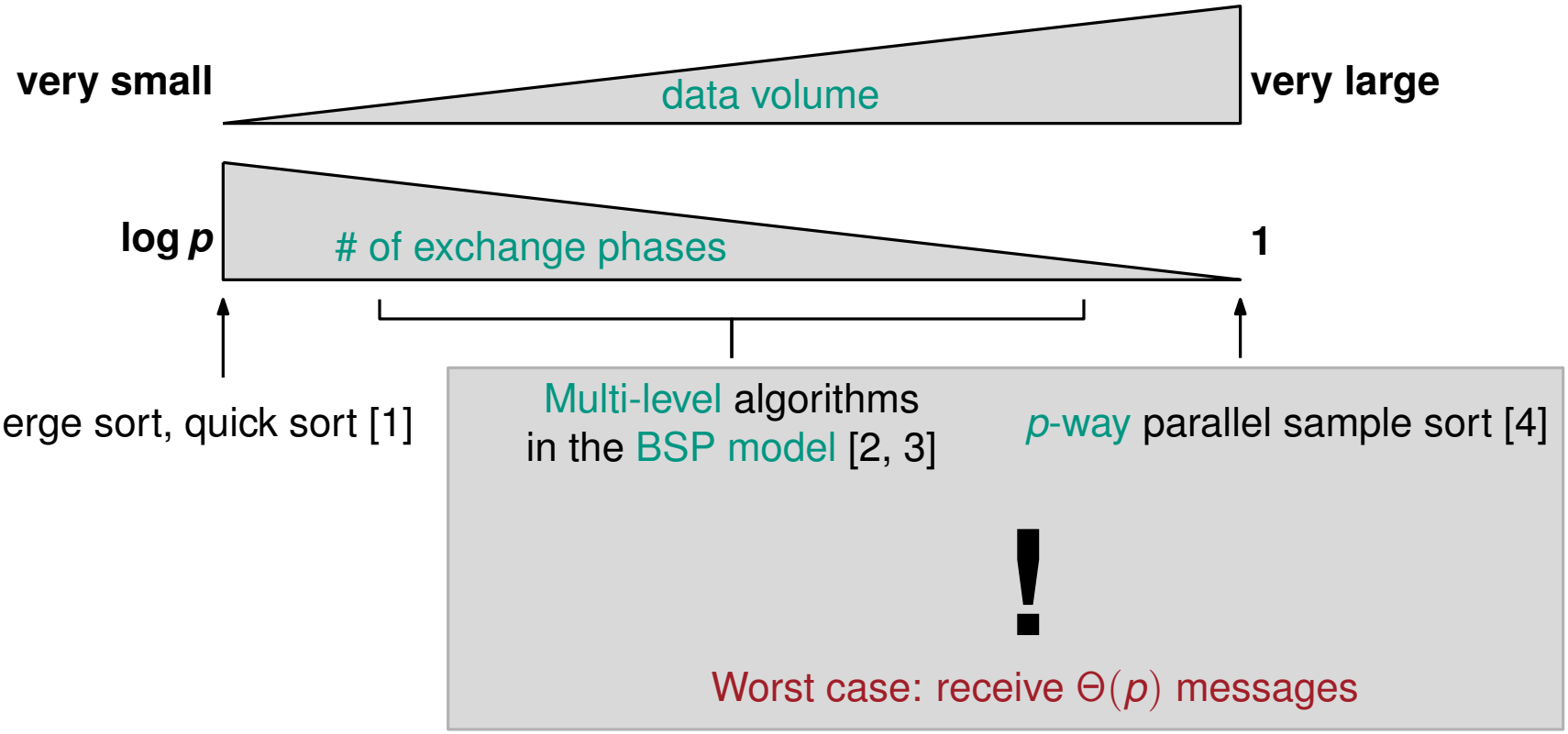
G. E. Blelloch et al. *3rd SPAA*

BSP Model

- Bulk synchronous
- Data exchange: p startups in practice



Massively Parallel Sorting Algorithms



[1] J. Jaja. *An Introduction to Parallel Algorithms*, 1992

[2] A. Gerbessiotis and L. Valiant. *JPDC*, 1994

[3] M. T. Goodrich. *SICOMP*, 1999

[4] G. E. Blelloch et al. *3rd SPAA*, 1991

Model of Computation

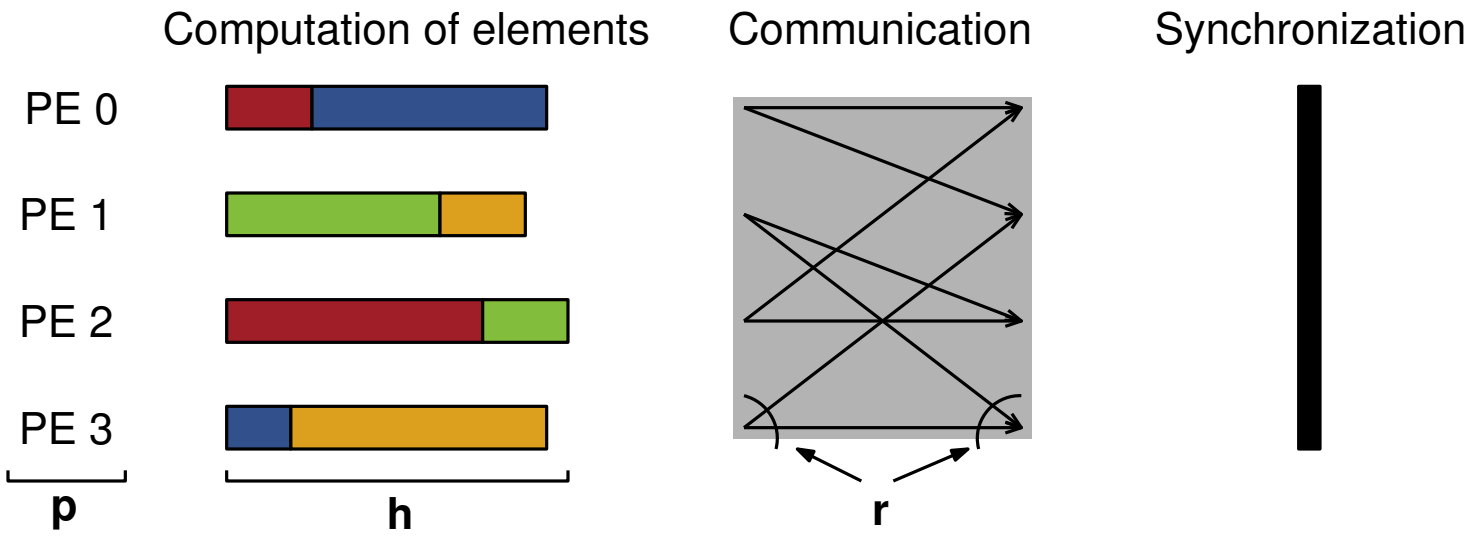
BSP model generalization

- Data exchange function: $\text{Exch}(p, h, r)$

involved PEs

max send/receive volume per PE

send/recv messages per PE



Comparison

Assumptions

- Number of levels $k \in \mathcal{O}(1)$
- Single-ported message passing
  - Sending of ℓ machine words: $\alpha + \beta\ell$

Algorithm	Isoefficiency function
p-way parallel sample sort [1]	$\mathcal{O}(p^2 \cdot \frac{1}{\log p})$
Multi-level BSP-based [2,3]	$\Omega(p^2 \cdot \frac{1}{\log p})$ in our model

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Multi-level BSP-based [2,3]	$\Omega(p^2 \cdot \frac{1}{\log p})$ in our model
Multi-level merge sort	$\mathcal{O}(p^{1+\frac{1}{k}} \cdot \log p)$
Multi-level sample sort	$\mathcal{O}(p^{1+\frac{1}{k}} \cdot \frac{1}{\log p})$

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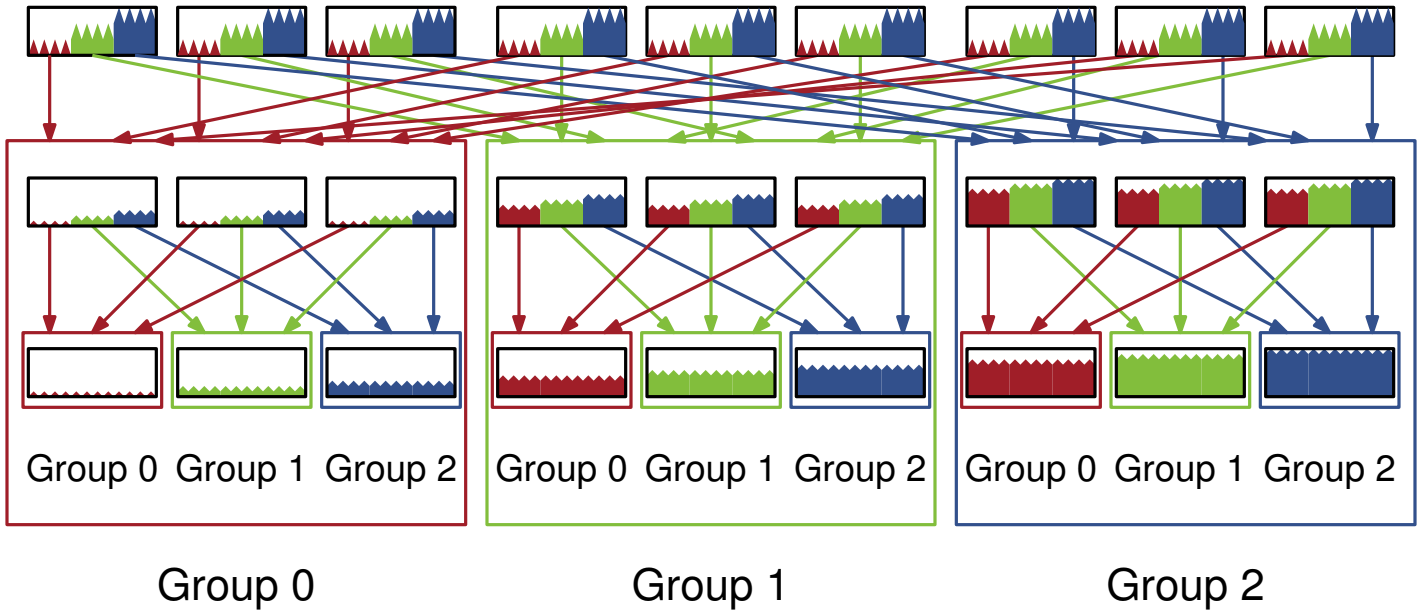
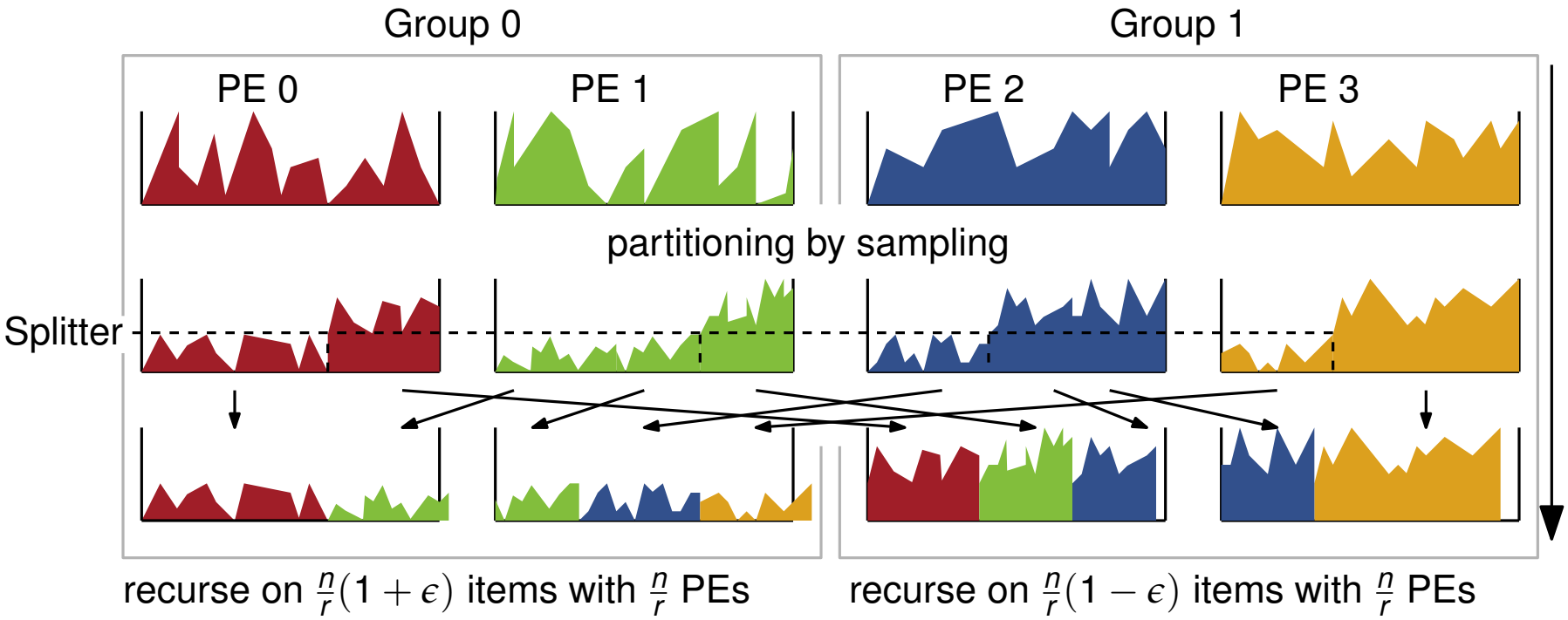
Multi-Level Sorting Approach

- Subdivide PEs into groups
- Move data to suitable group
- k levels of recursion
  - Groups $r \approx \sqrt[k]{p}$



Adaptive Multi-Level Sample Sort



Requirements

- Fast parallel sorting of samples
- Sample reduction by overpartitioning
- Reduce startup overheads to $\mathcal{O}(k\sqrt[k]{p})$

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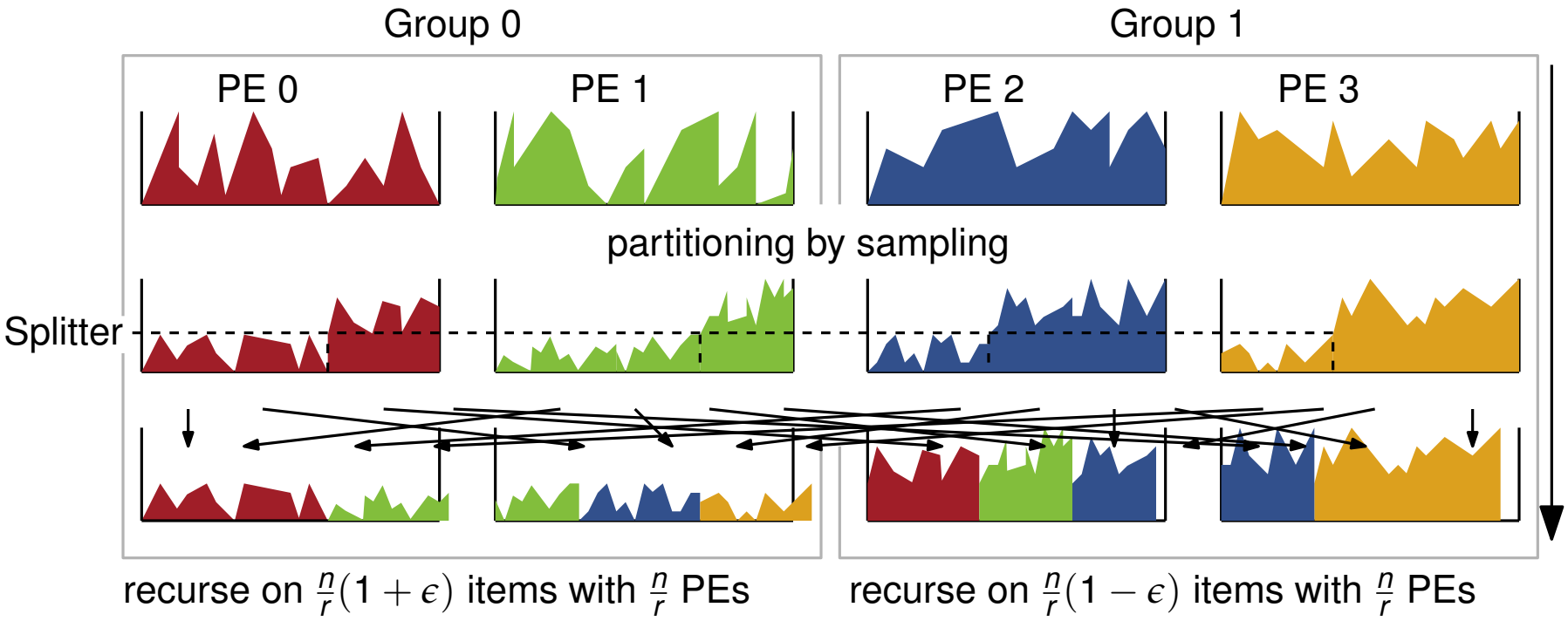

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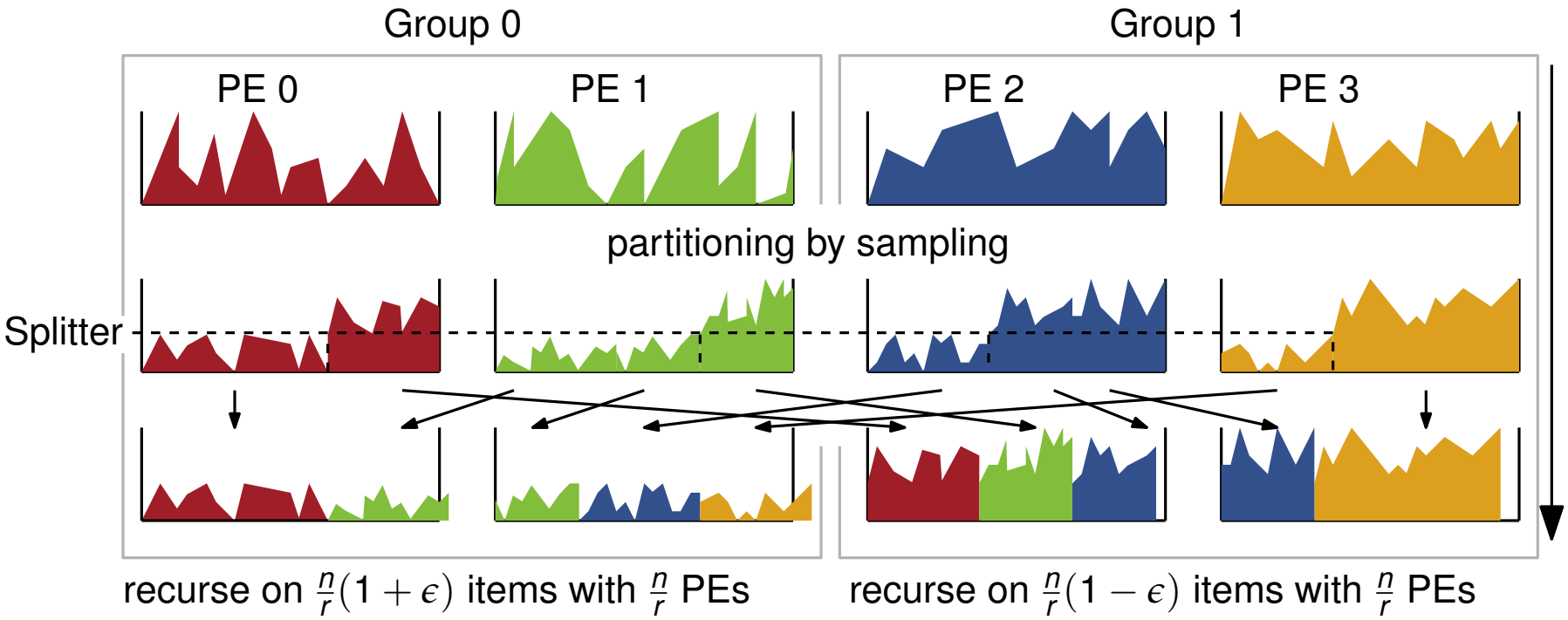

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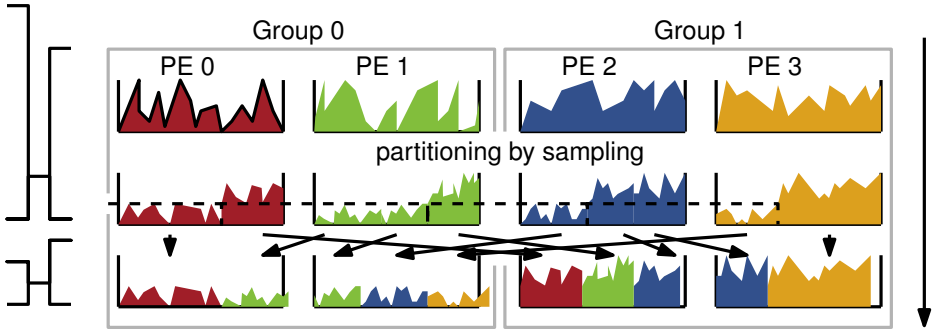
Adaptive Multi-Level Sample Sort

Open submodules

1. Fast sample sorting
   - Oversampling
2. Optimal overpartitioning
3. Group-based data delivery



Fast Parallel Sample Sorting

- Parallel sorting of s samples
- Rectangular $a \times b$ array of PEs

PE 0	PE 1	PE 2
v x	**d t**	**h g**
PE 3	PE 4	PE 5
o q	**r i**	**l w**
PE 6	PE 7	PE 8
f m	**p c**	**b k**

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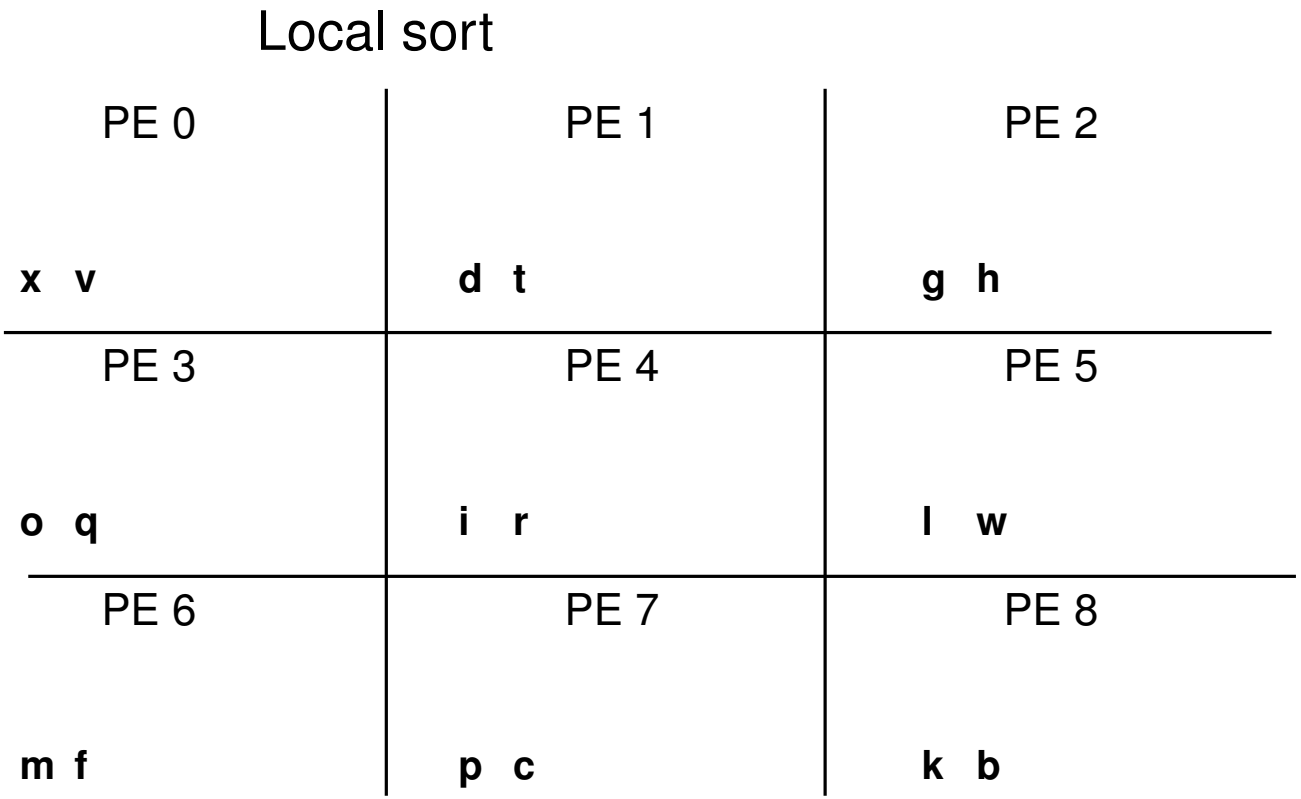

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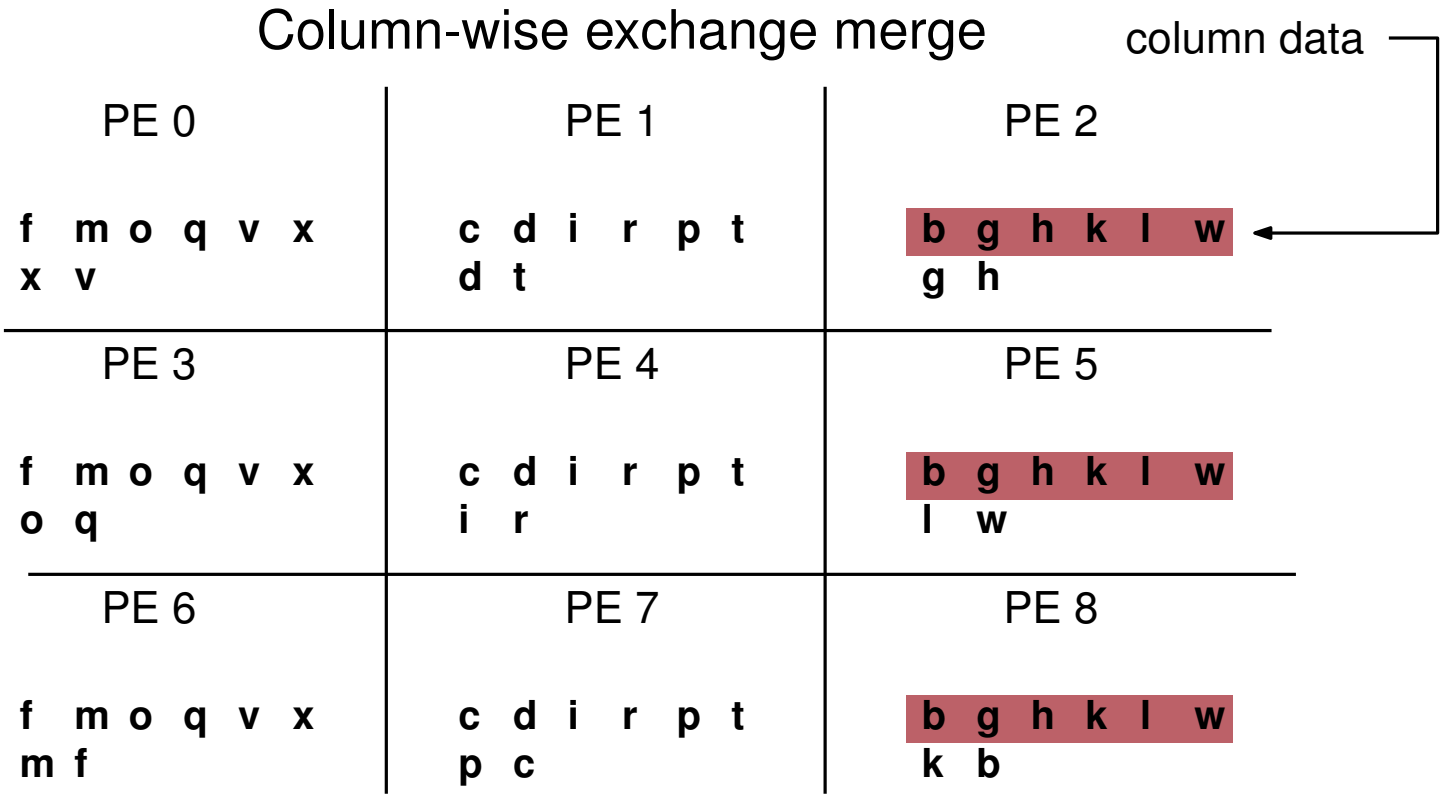

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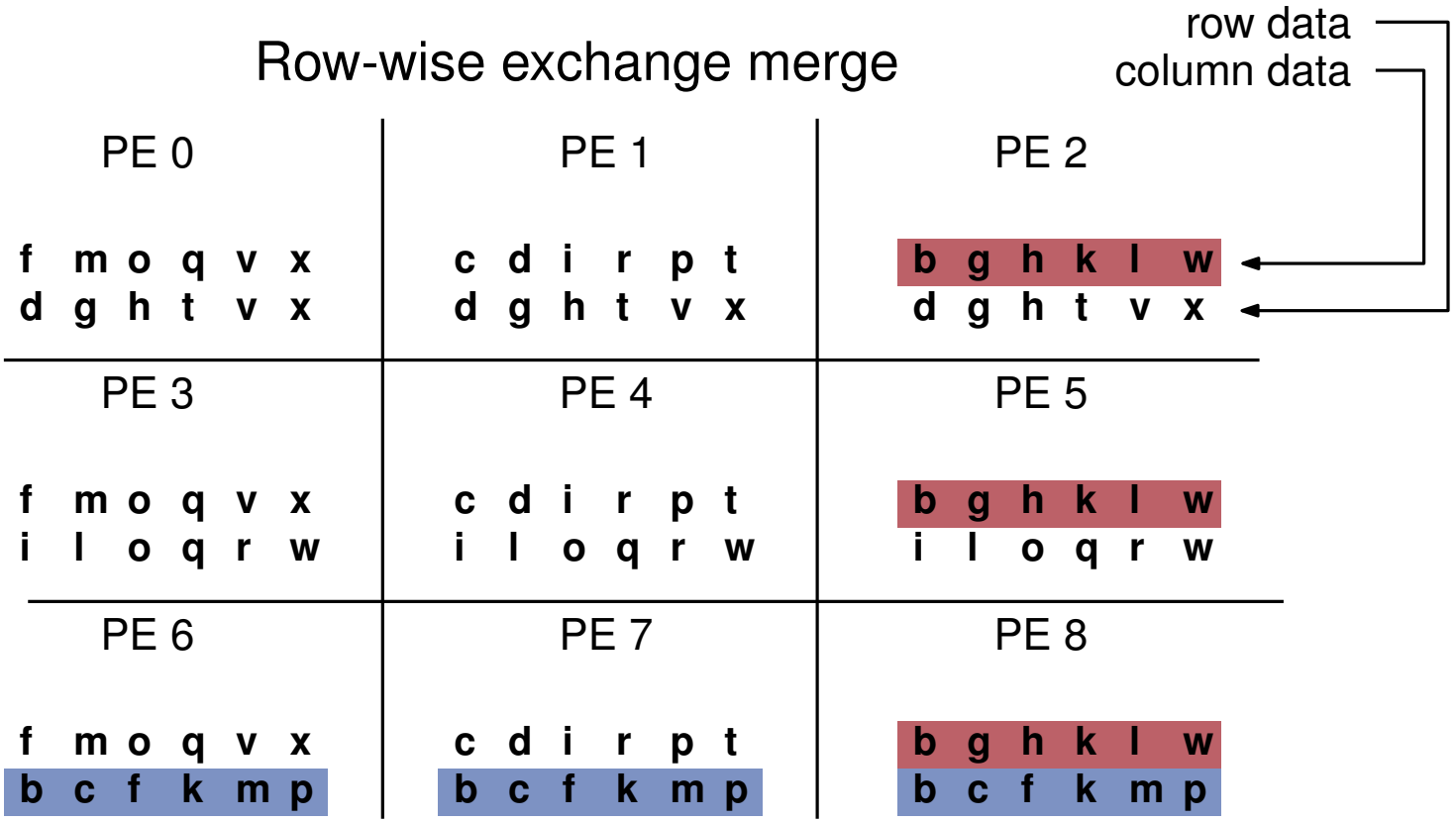

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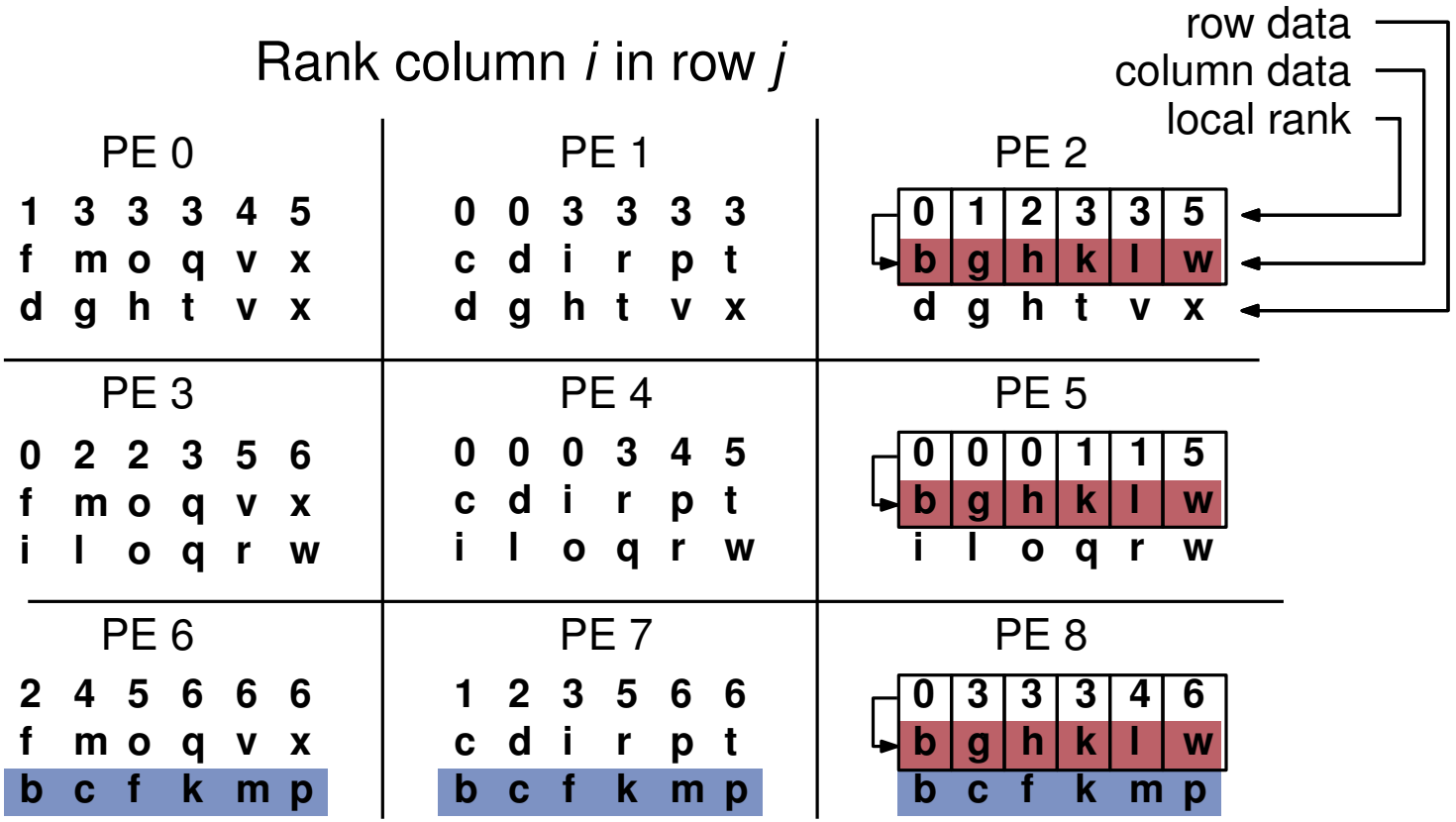

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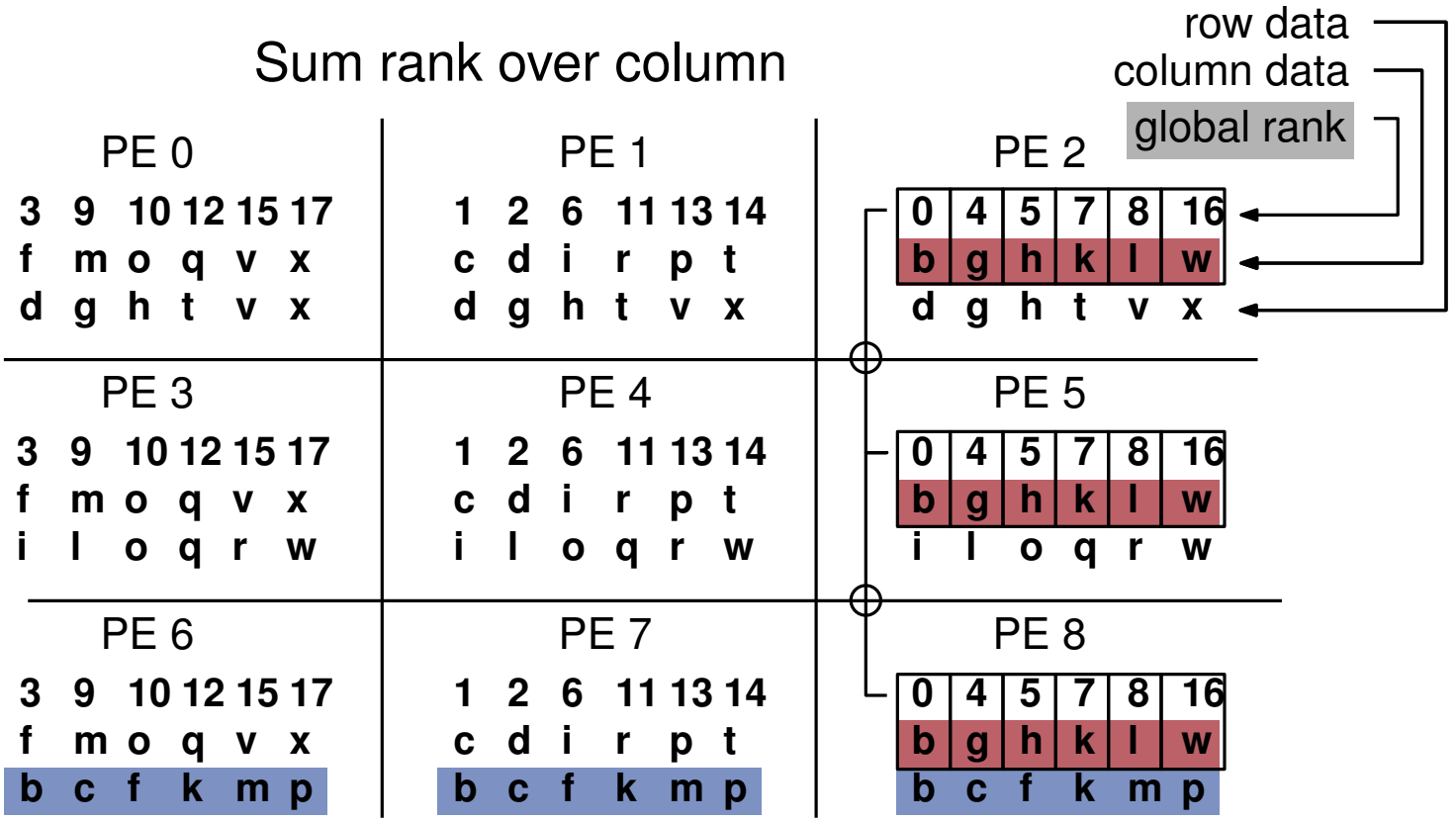

Fast Parallel Sample Sorting

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Single-ported message passing

- Sending of ℓ machine words: $\alpha + \beta\ell$

- Local sort
- Column-wise allgather merge
- Row-wise allgather merge
- Rank column *i* in row *j*
- Sum rank over column

$$\mathcal{O}(\alpha \log p + \beta \frac{s}{\sqrt{p}} + \frac{s}{p} \log \frac{s}{p})$$

Adaptive Multi-Level Sample Sort

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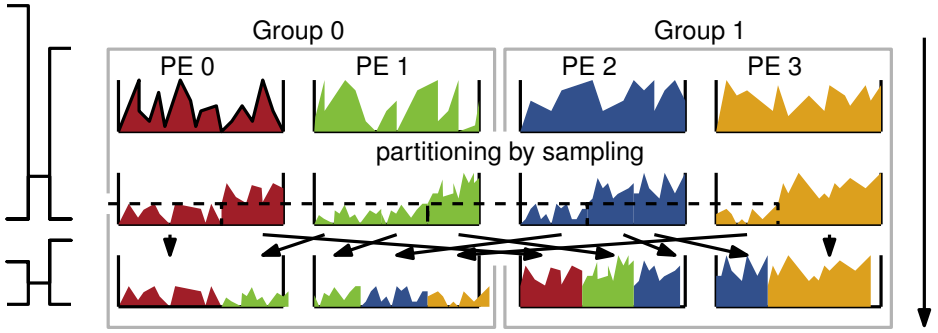
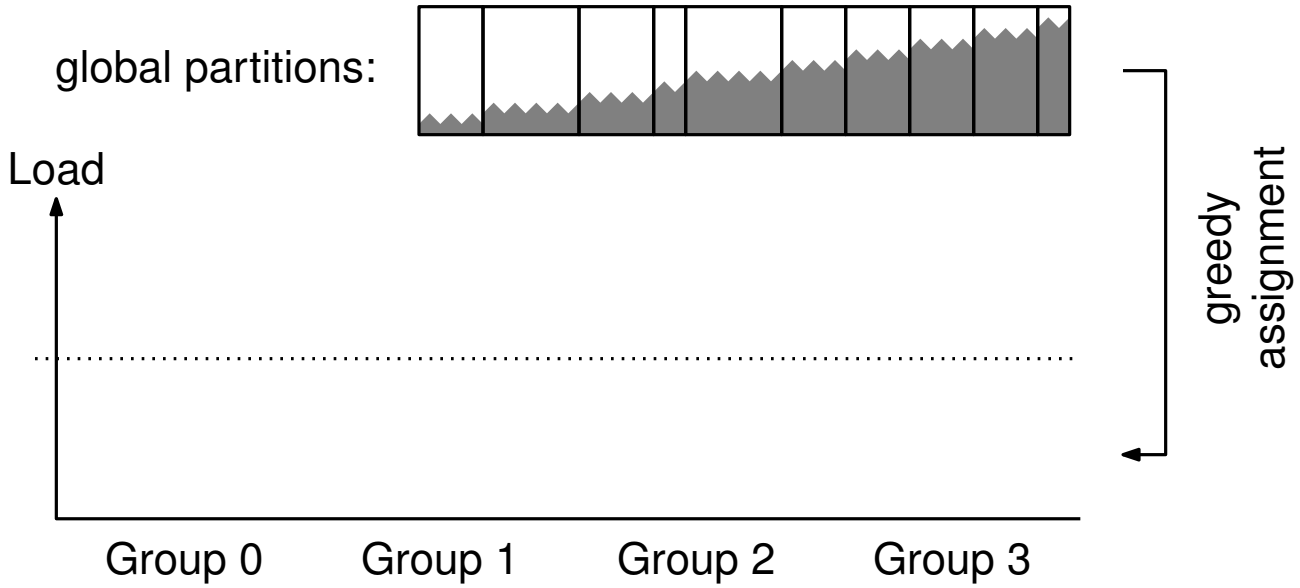


Optimal Overpartitioning

- Requirement: $L_{max} = (1+\epsilon)\frac{n}{r}$ with high probability
- Oversampling: a
- Overpartitioning: $b \in \Theta(\frac{1}{\epsilon})$



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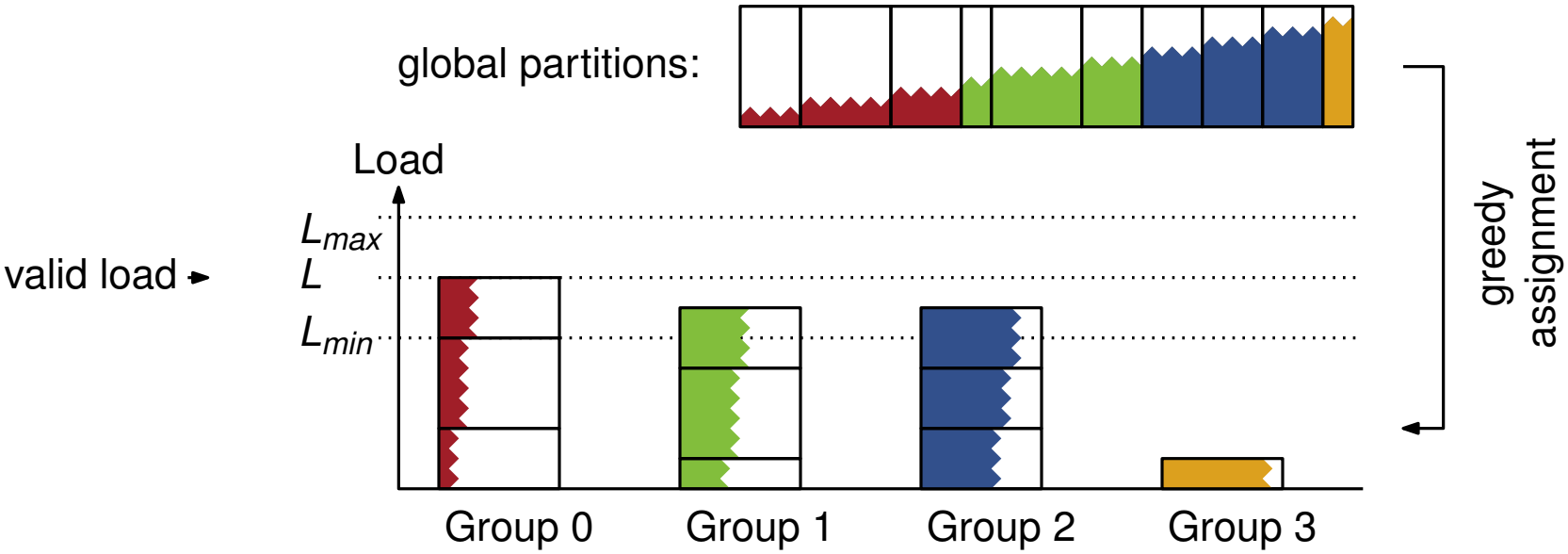

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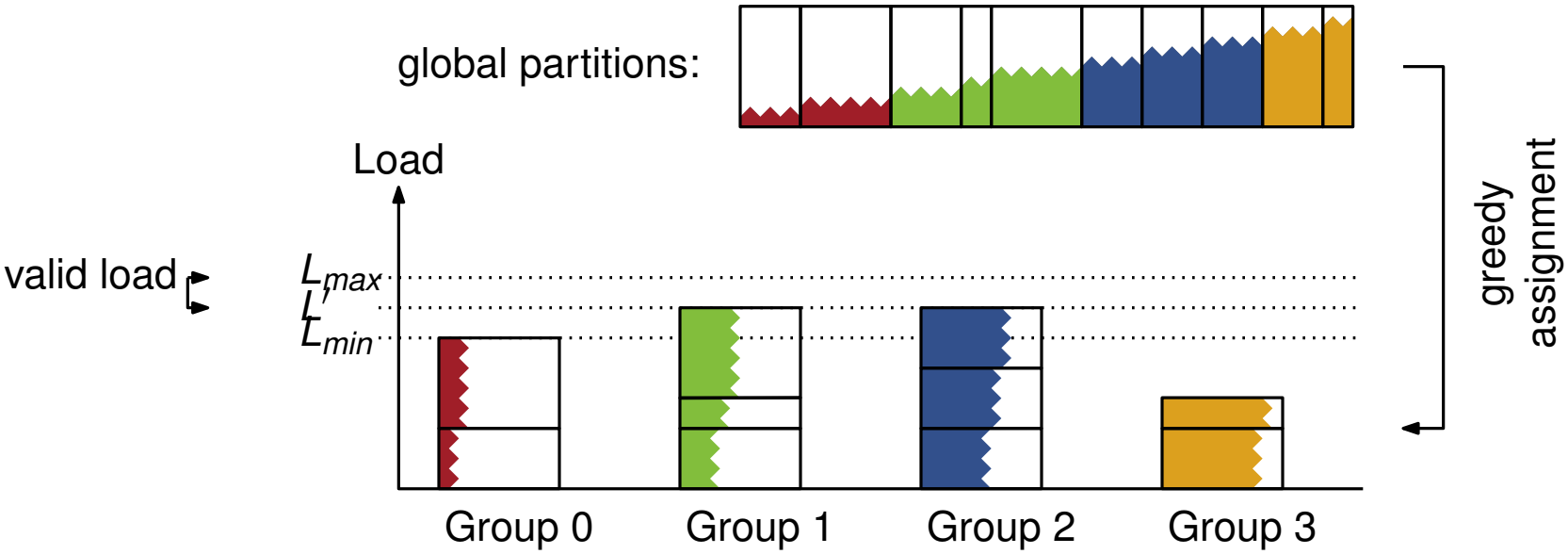

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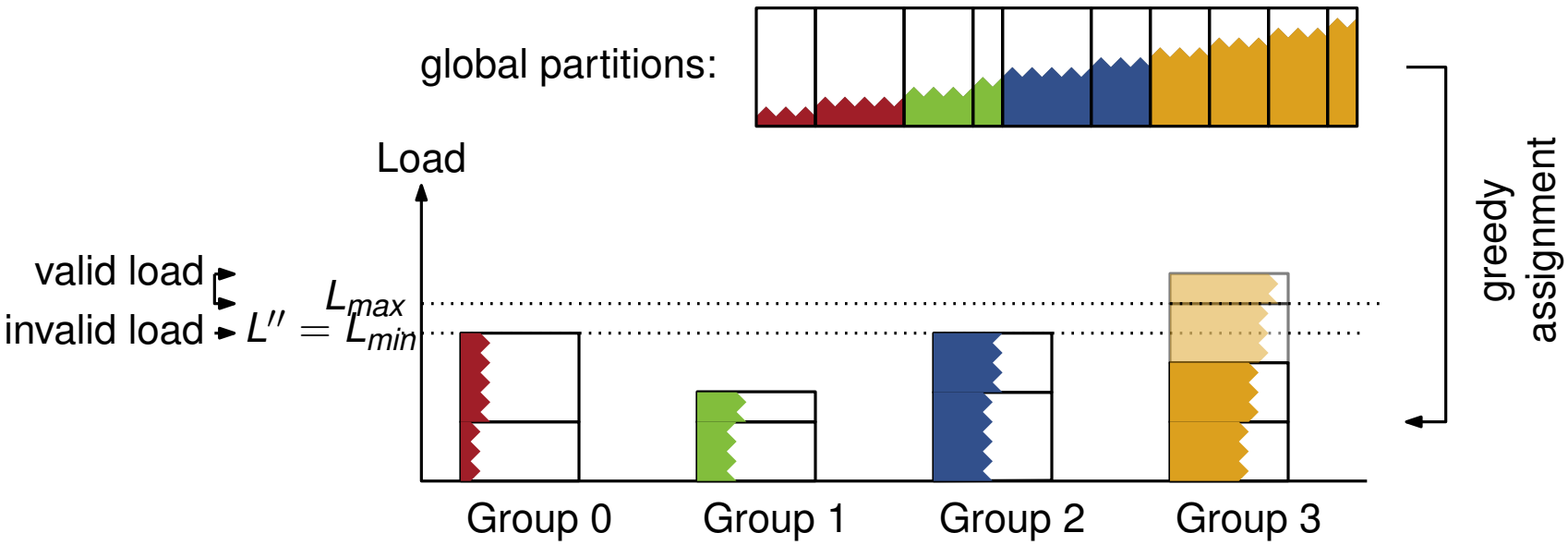
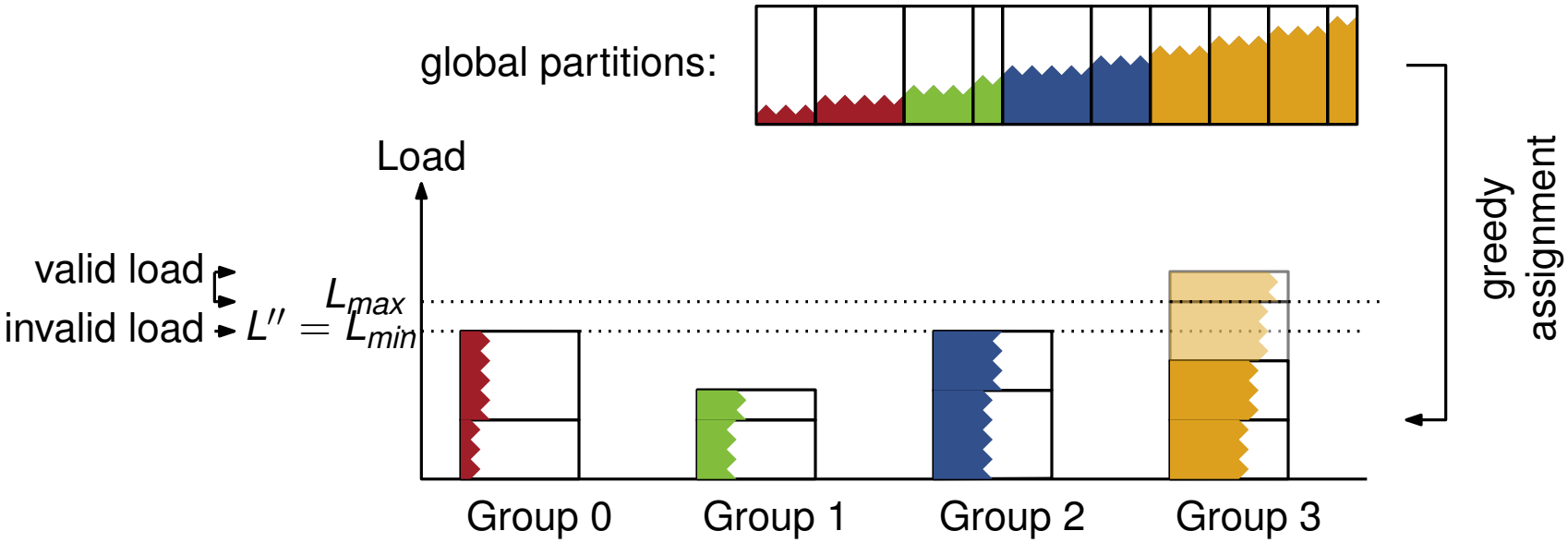


Optimal Overpartitioning

- Requirement: $L_{max} = (1+\epsilon)\frac{n}{r}$ with high probability
- Oversampling: a
- Overpartitioning: $b \in \Theta(\frac{1}{\epsilon})$
- Fewer samples $abr \in \Theta(r \log r)$

$$\mathcal{O}(br + \alpha \log p)$$



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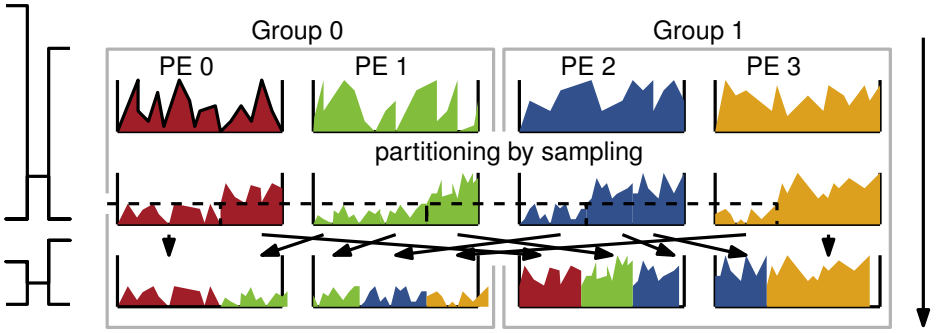


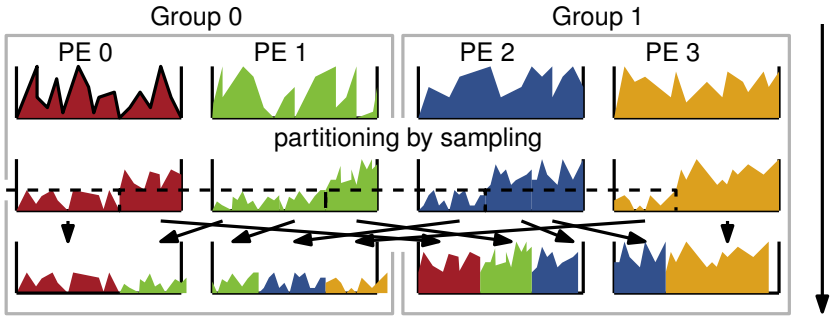
Group-Based Data Delivery

Goal

- Partition i to group i
- Each PE in group receives same amount of data
- $(1 + o(1))\text{Exch}(p, \frac{n}{p}, \mathcal{O}(r))$

  - p: involved PEs
  - $\frac{n}{p}$: max send/receive volume per PE
  - $\mathcal{O}(r)$: send/recv messages per PE

- Reduce startup overheads to $\mathcal{O}(\sqrt[k]{p})$



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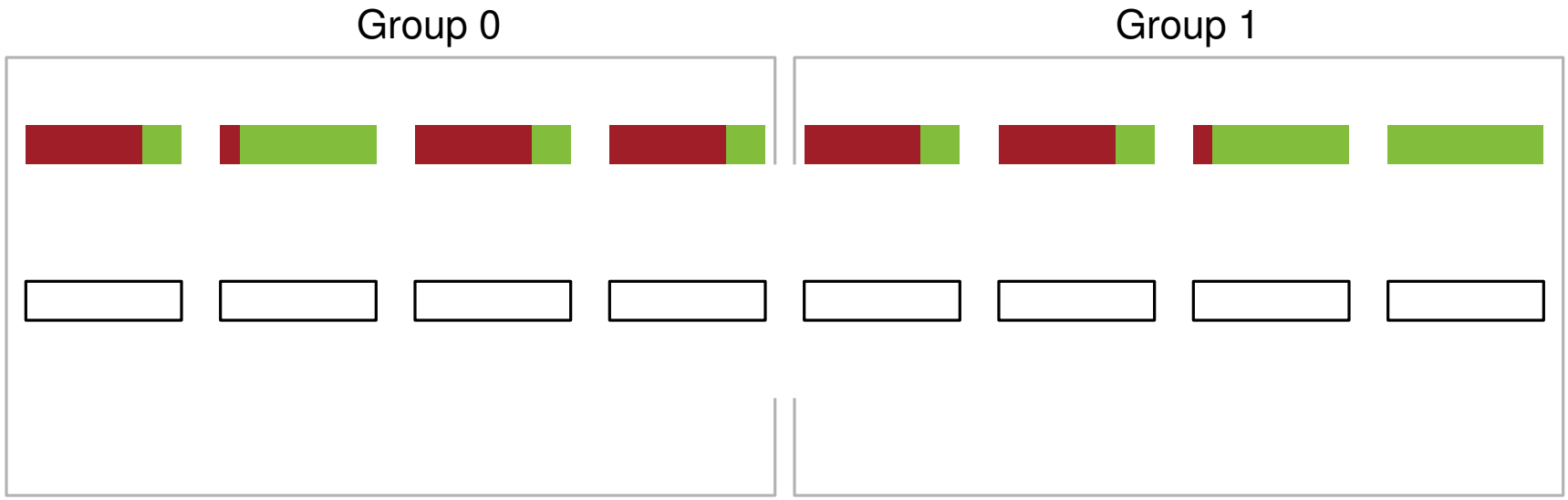
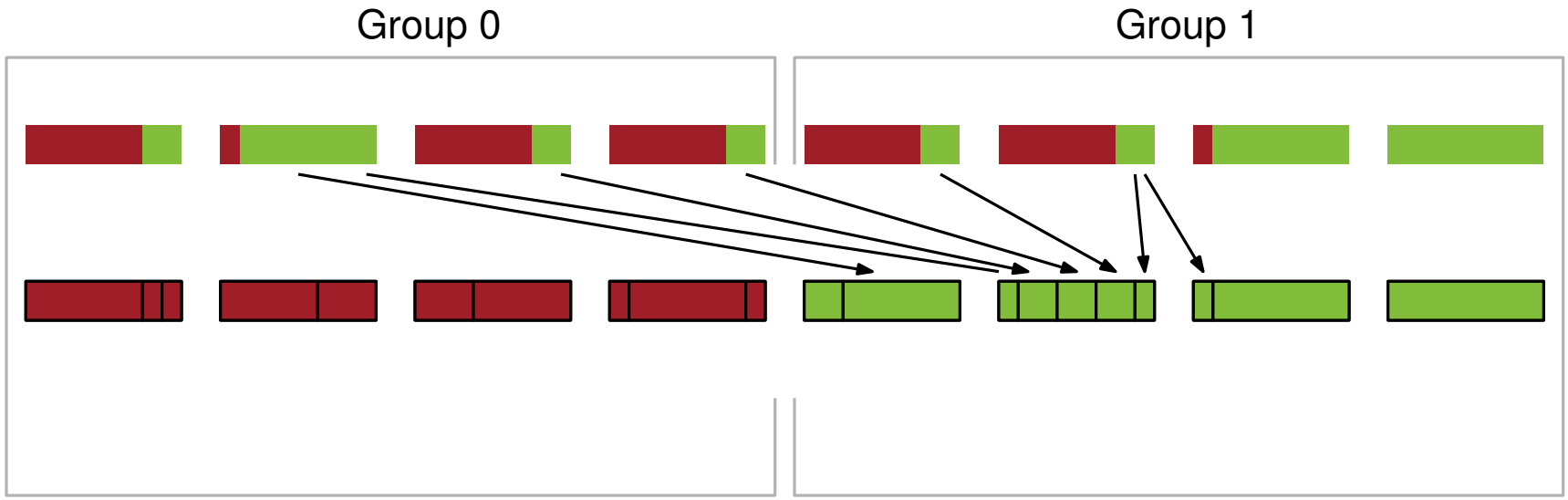


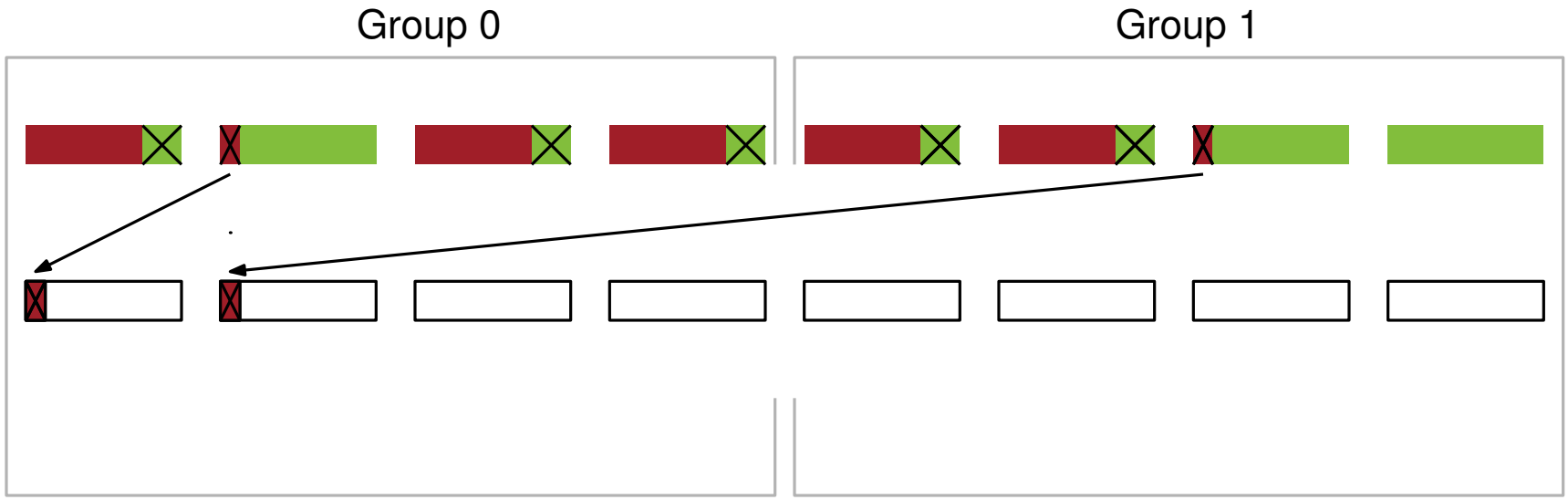
Group-Based Data Delivery

Trivial approach



Group-Based Data Delivery

Our approach



- Distribution of small pieces $|\cdot| \leq \frac{n}{2pr}$ // round-robin

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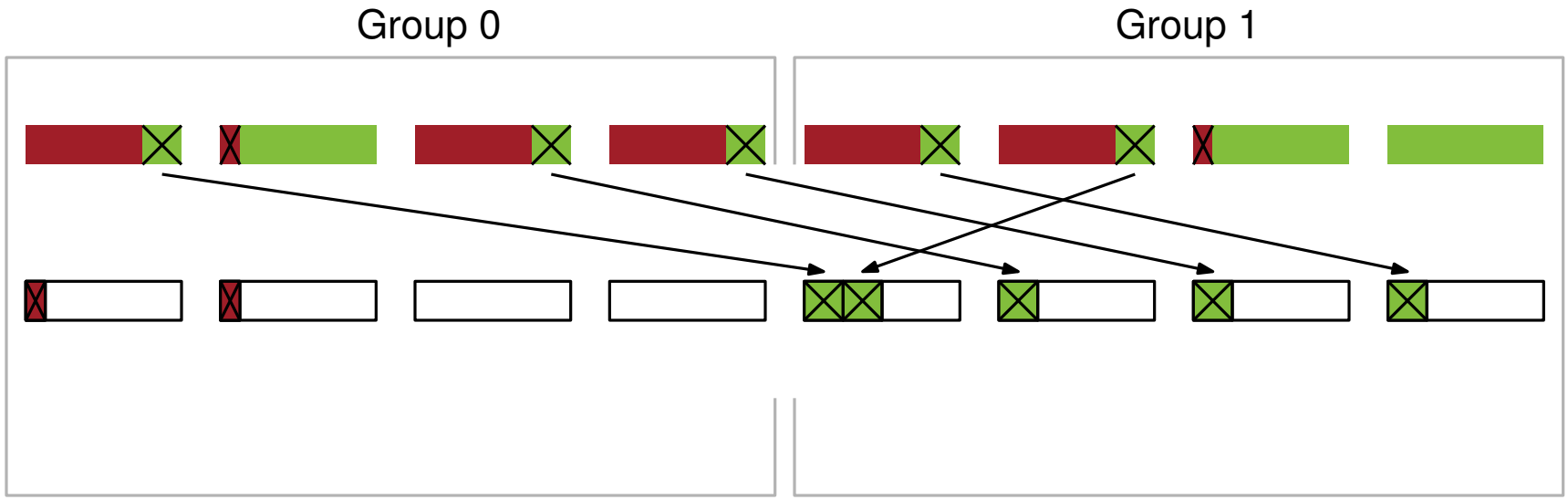

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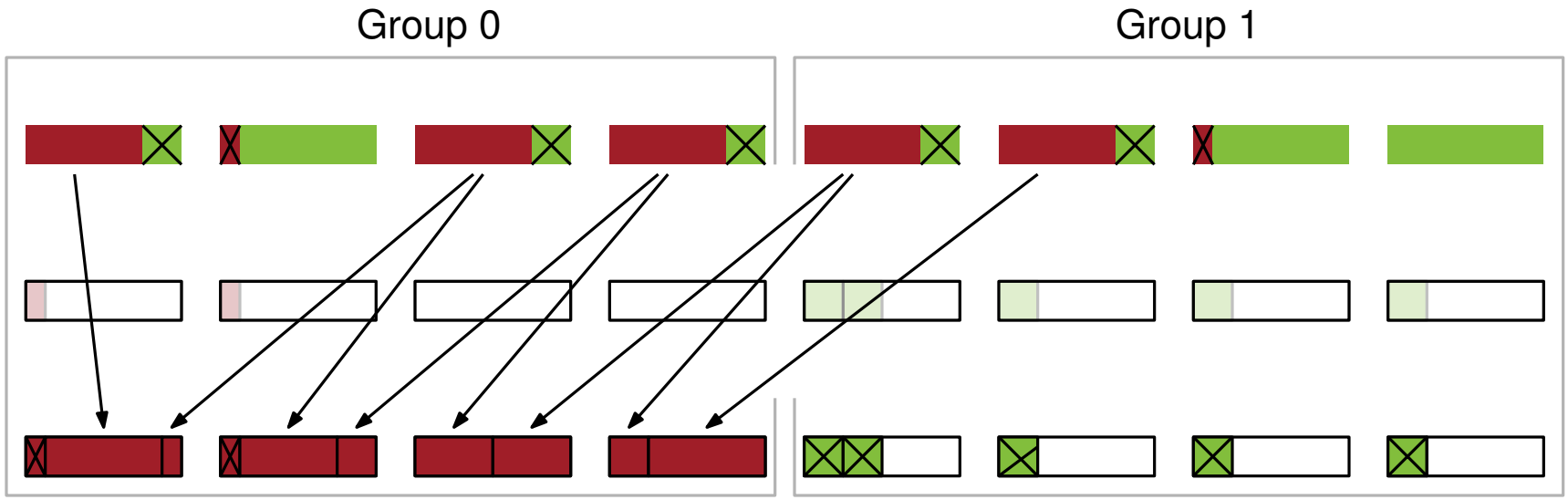

- Distribution of small pieces $|\cdot| \leq \frac{n}{2pr}$ // round-robin
- Distribution of large pieces // prefix-sum and merging

Group-Based Data Delivery

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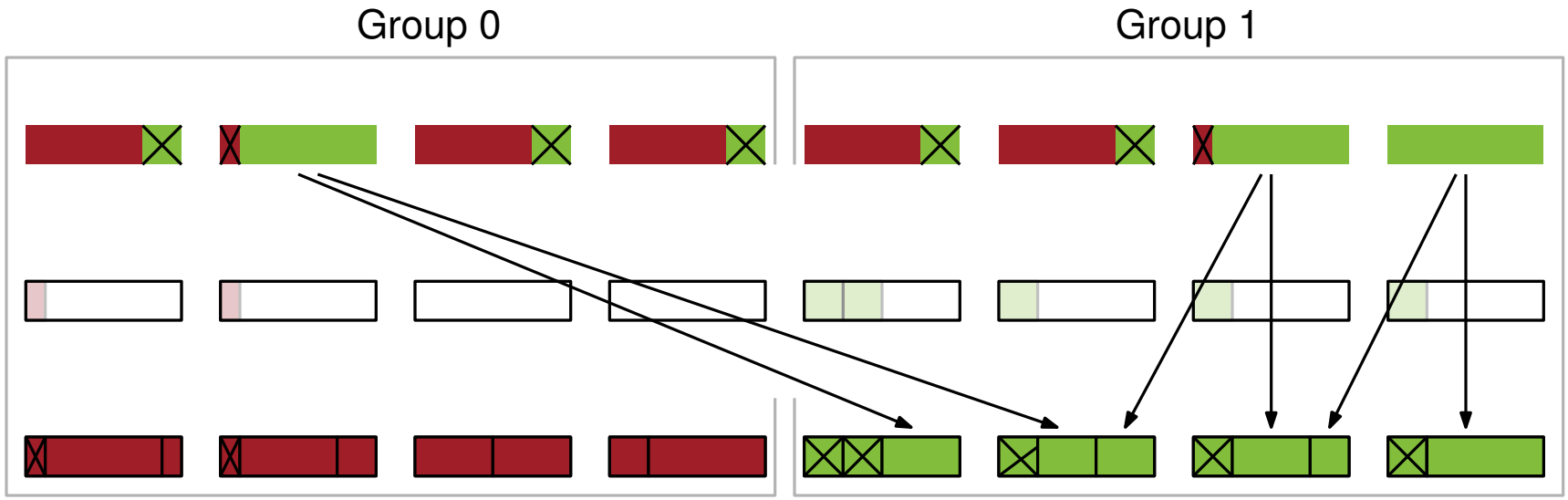

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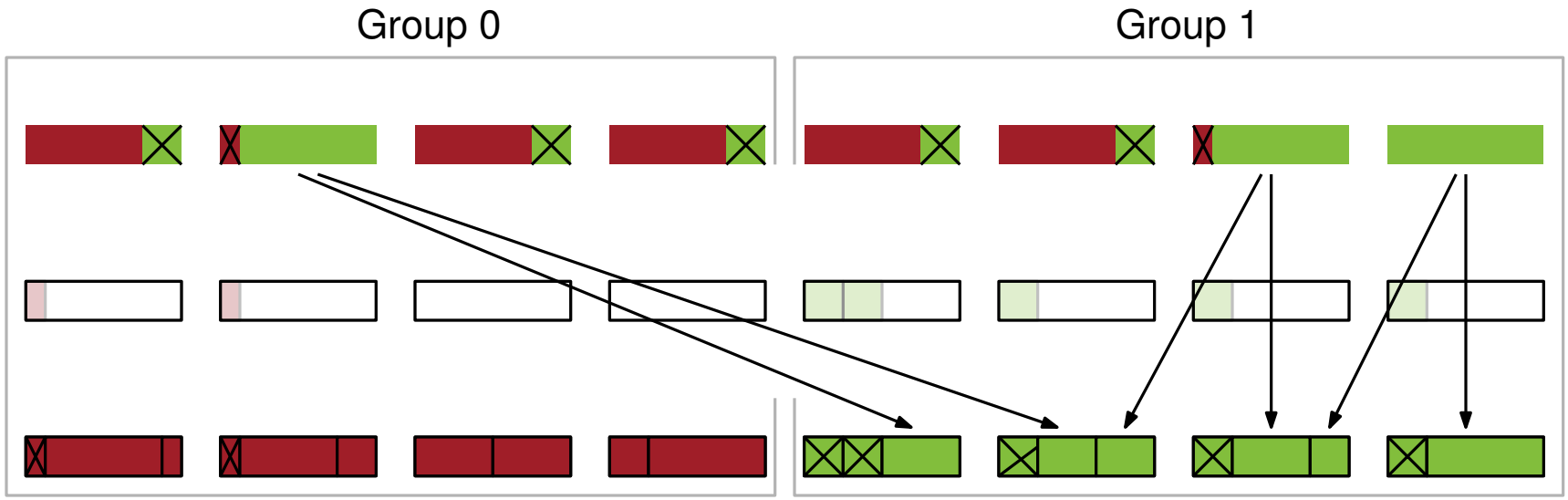

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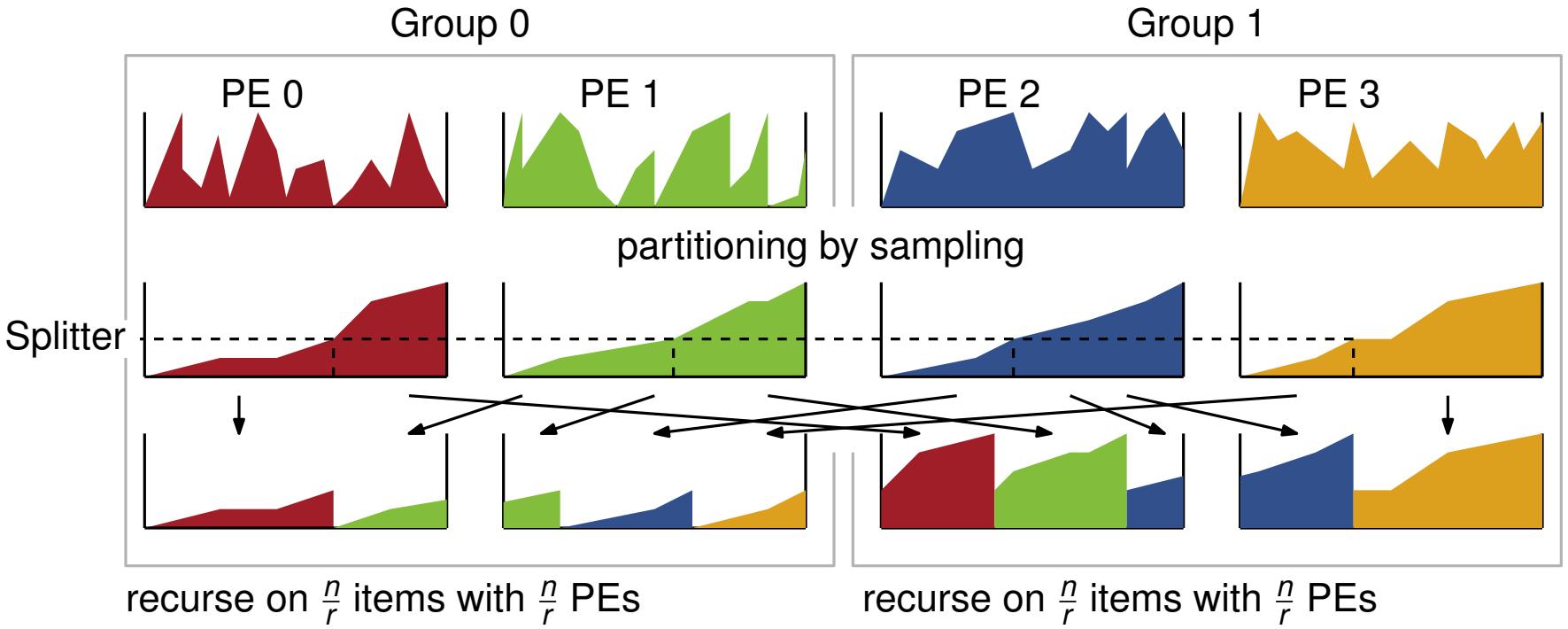
- Distribution of large pieces // prefix-sum and merging

Reduces startup overheads to $\mathcal{O}(\sqrt[k]{p})$

Recurse Last Multiway Mergesort

Highlights

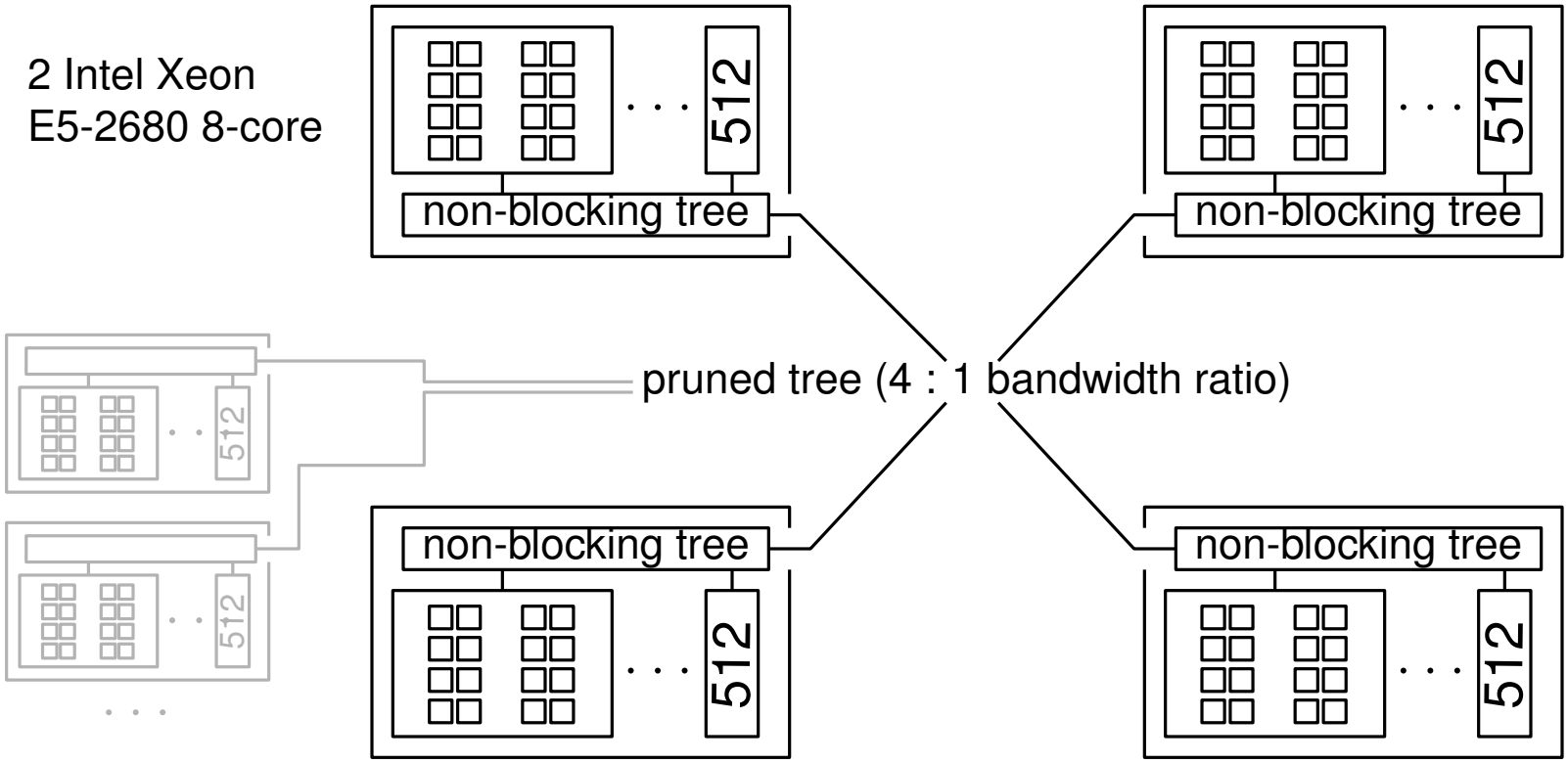
- Multisequence selection
- Perfect load balance
- Reduces startup overheads to $\mathcal{O}(k\sqrt[k]{p})$



Experiments

System

SuperMUC in Munich



Cores used: 32 768 (4 islands)

Experiments

Sample sort median wall-times in seconds

n/p	p = 512	2 048	8 192	32 768
10^5	0.0228	0.0277	0.0359	0.0707
10^6	0.2212	0.2589	0.2687	0.9171
10^7	2.6523	2.9797	4.0625	6.0932

Speedup of sample sort compared to sequential sort

n/p	p = 512	2 048	8 192	32 768
10^5	273	956	3 208	6 929
10^6	321	1 146	4 747	6 164
10^7	295	1 124	–	–

Level 1 Level 2 Level 3

Comparison to Literature

Solomonik and Kale [1]: CrayXT 4

- Slower processors, higher bandwidth
- $n = 8 \cdot 10^6 p$, up to $p = 2^{15}$ // vs. $n_{\text{ref}} = 10^7 p$
  - Similar performance

MP-sort [2]: Cray XE6

- $n = 10^5 p$ and $p = 2^{14}$ // vs. $p_{\text{ref}} = 2^{15}$
  - 289 times faster
- 6 times faster for large inputs

[1] Solomonik and Kale. *IPDPS 2010*

[2] Y. Feng et al. *2014*

Conclusion

Result

- Scalable in theory and practice
- Improved wall-time: large p and moderate n
- Competitive: large p and large n

Future work

- Perform experiments with more PEs
- Shared memory on node-local level
- Better exchange algorithms
- Fault tolerance

Acknowledgement

The authors gratefully acknowledge the Gauss Centre for Supercomputing e.V. (www.gauss-centre.eu) for funding this project by providing computing time on the GCS Supercomputer SuperMUC at Leibniz Supercomputing Centre (LRZ, www.lrz.de).