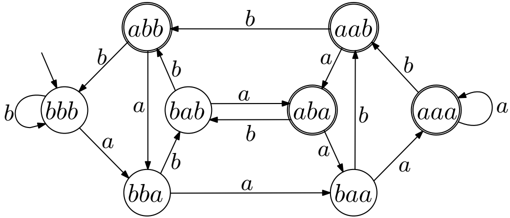
3. Übung – TGI

Lorenz Hübschle-Schneider, Tobias Maier

INSTITUT FÜR THEORETISCHE INFORMATIK, PROF. SANDERS

Minimalautomaten



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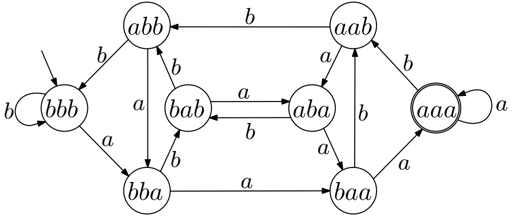

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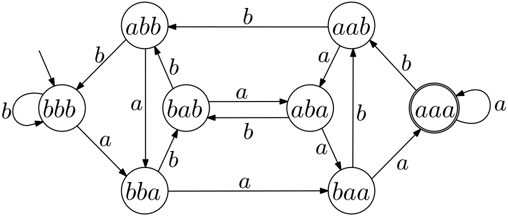

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baa								
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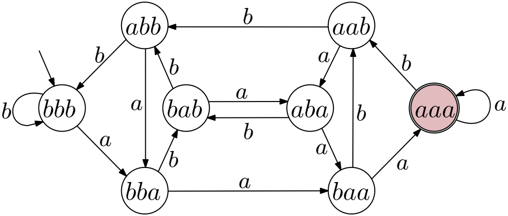
aab								
aaa								

Minimalautomaten



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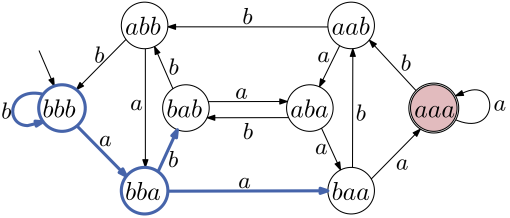
bba								
bab								
abb								
baa								
aba								
aab								
aaa	ε	ε	ε	ε	ε	ε	ε	

Minimalautomaten



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bab								
abb								
baa								
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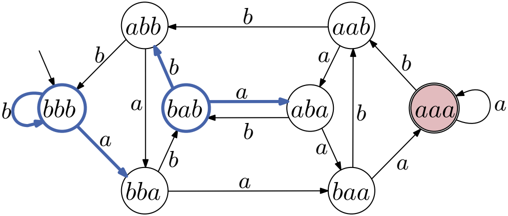
aaa	ε	ε	ε	ε	ε	ε	ε	

Minimalautomaten



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bab								
abb								
baa								
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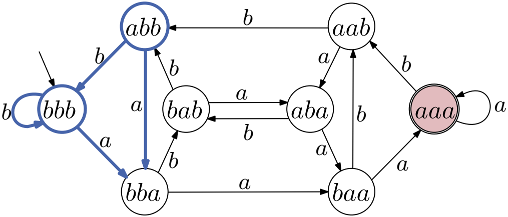
aaa	ε	ε	ε	ε	ε	ε	ε	

Minimalautomaten



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bba								
bab								
abb								
baa								
aba								
aab								
aaa	ε	ε	ε	ε	ε	ε	ε	

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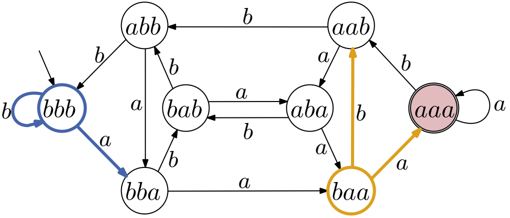

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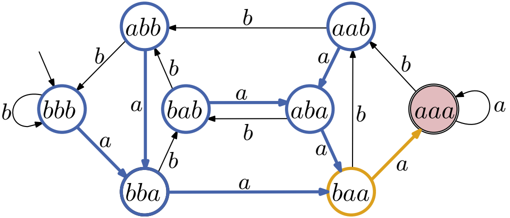
bbb								
bba								
bab								
abb								
baa	$a \cdot \varepsilon$							
aba								
aab								
aaa	ε	ε	ε	ε	ε	ε	ε	

Minimalautomaten



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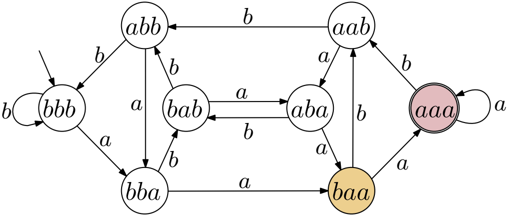
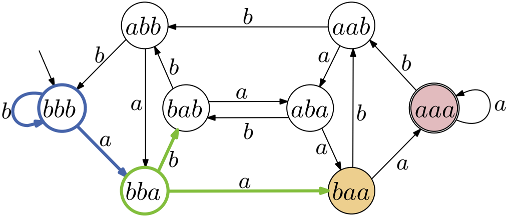
bba								
bab								
abb								
baa	a	a	a	a				
aba					a			
aab					a			
aaa	ε	ε	ε	ε	ε	ε	ε	

Minimalautomaten



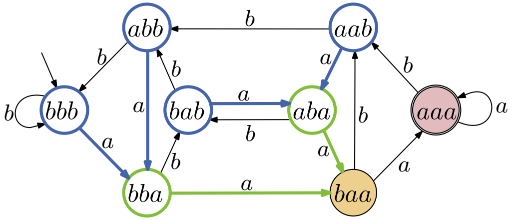
	bbb	bba	bab	abb	baa	aba	aab	aaa
bbb								
bba								
bab								
abb								
baa	a	a	a	a				
aba					a			
aab					a			
aaa	ε	ε	ε	ε	ε	ε	ε	

Minimalautomaten



	bbb	bba	bab	abb	baa	aba	aab	aaa
bbb								
bba	$a \cdot a$							
bab								
abb								
baa	a	a	a	a				
aba					a			
aab					a			
aaa	ε	ε	ε	ε	ε	ε	ε	

Minimalautomaten



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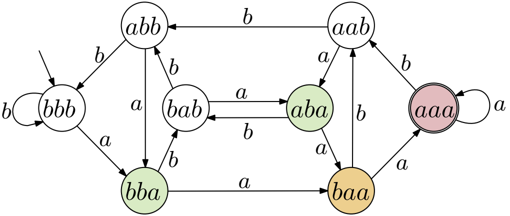
bbb								
bba	aa							
bab		aa						
abb		aa						
baa	a	a	a	a				
aba	aa		aa	aa	a			
aab		aa			a	aa		
aaa	ε	ε	ε	ε	ε	ε	ε	

Minimalautomaten



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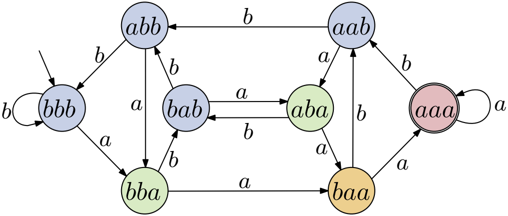
bbb								
bba	aa							
bab		aa						
abb		aa						
baa	a	a	a	a				
aba	aa		aa	aa	a			
aab		aa			a	aa		
aaa	ε	ε	ε	ε	ε	ε	ε	

Minimalautomaten



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bbb								
bba	aa							
bab	X	aa						
abb	X	aa	X					
baa	a	a	a	a				
aba	aa	X	aa	aa	a			
aab	X	aa	X	X	a	aa		
aaa	ε	ε	ε	ε	ε	ε	ε	

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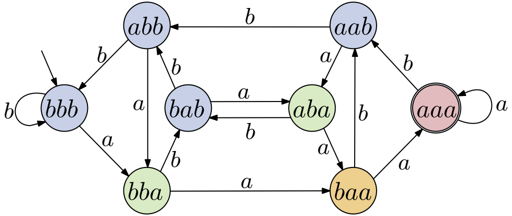

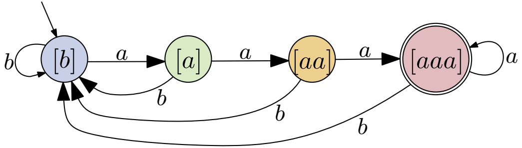

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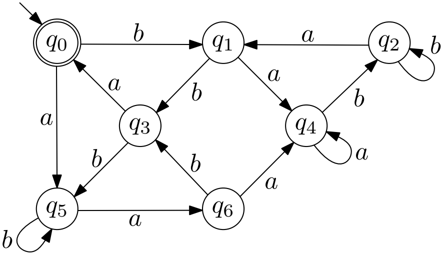

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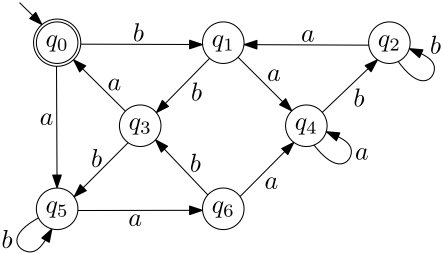
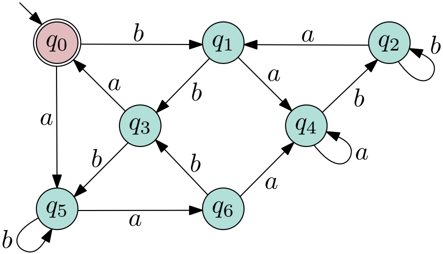


	q_0	q_1	q_2	q_3	q_4	q_5	q_6
q_0							
q_1							
q_2							
q_3							
q_4							
q_5							
q_6							

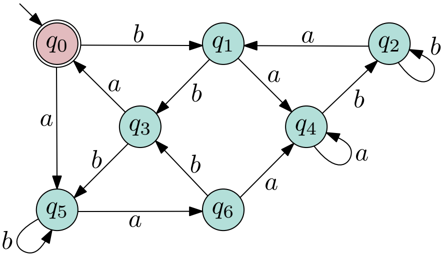
Minimalautomaten

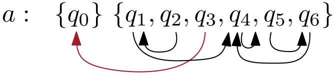


$a: \quad \{q_0\}\ \{q_1, q_2, q_3, q_4, q_5, q_6\}$

	q_0	q_1	q_2	q_3	q_4	q_5	q_6
q_0							
q_1	ε						
q_2	ε						
q_3	ε						
q_4	ε						
q_5	ε						
q_6	ε						

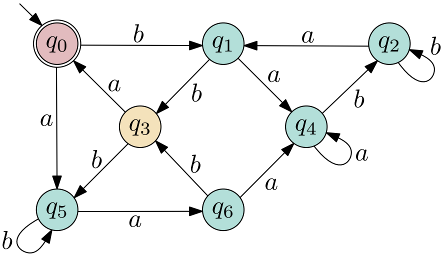
Minimalautomaten





	q_0	q_1	q_2	q_3	q_4	q_5	q_6
q_0							
q_1	ε						
q_2	ε						
q_3	ε						
q_4	ε						
q_5	ε						
q_6	ε						

Minimalautomaten

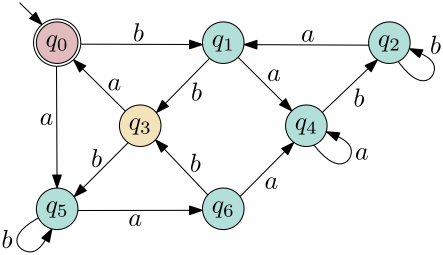


$a: \quad \{q_0\}\ \{q_1, q_2, q_3, q_4, q_5, q_6\}$

$b: \quad \{q_0\}\ \{q_3\}\ \{q_1, q_2, q_4, q_5, q_6\}$

	q_0	q_1	q_2	q_3	q_4	q_5	q_6
q_0							
q_1	ε						
q_2	ε						
q_3	ε	a	a				
q_4	ε			a			
q_5	ε			a			
q_6	ε			a			

Minimalautomaten

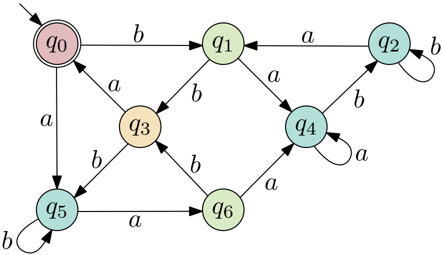


$a: \quad \{q_0\} \; \{q_1, q_2, q_3, q_4, q_5, q_6\}$

$b: \quad \{q_0\} \; \{q_3\} \; \{q_1, q_2, q_4, q_5, q_6\}$

	q_0	q_1	q_2	q_3	q_4	q_5	q_6
q_0							
q_1	ε						
q_2	ε						
q_3	ε	a	a				
q_4	ε			a			
q_5	ε			a			
q_6	ε			a			

Minimalautomaten



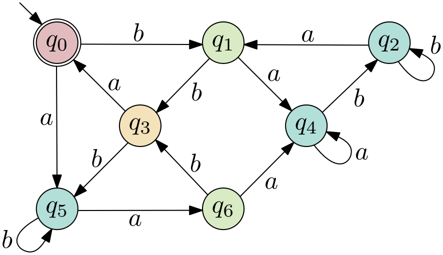
$a:\quad \{q_0\}\ \{q_1, q_2, q_3, q_4, q_5, q_6\}$

$b:\quad \{q_0\}\ \{q_3\}\ \{q_1, q_2, q_4, q_5, q_6\}$

$a:\quad \{q_0\}\ \{q_3\}\ \{q_1, q_6\}\ \{q_2, q_4, q_5\}$

	q_0	q_1	q_2	q_3	q_4	q_5	q_6
q_0							
q_1	ε						
q_2	ε	ba					
q_3	ε	a	a				
q_4	ε	ba		a			
q_5	ε	ba		a			
q_6	ε		ba	a	ba	ba	

Minimalautomaten



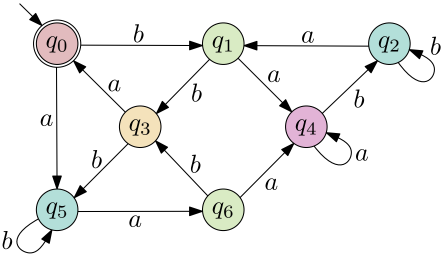
$a: \quad \{q_0\}\ \{q_1, q_2, q_3, q_4, q_5, q_6\}$

$b: \quad \{q_0\}\ \{q_3\}\ \{q_1, q_2, q_4, q_5, q_6\}$

$a: \quad \{q_0\}\ \{q_3\}\ \{q_1, q_6\}\ \{q_2, q_4, q_5\}$

	q_0	q_1	q_2	q_3	q_4	q_5	q_6
q_0							
q_1	ε						
q_2	ε	ba					
q_3	ε	a	a				
q_4	ε	ba		a			
q_5	ε	ba		a			
q_6	ε		ba	a	ba	ba	

Minimalautomaten



	q_0	q_1	q_2	q_3	q_4	q_5	q_6
q_0							
q_1	ε						
q_2	ε	ba					
q_3	ε	a	a				
q_4	ε	ba	aba	a			
q_5	ε	ba		a	aba		
q_6	ε		ba	a	ba	ba	

$a: \quad \{q_0\}\ \{q_1, q_2, q_3, q_4, q_5, q_6\}$

$b: \quad \{q_0\}\ \{q_3\}\ \{q_1, q_2, q_4, q_5, q_6\}$

$a: \quad \{q_0\}\ \{q_3\}\ \{q_1, q_6\}\ \{q_2, q_4, q_5\}$

$b: \quad \{q_0\}\ \{q_3\}\ \{q_1, q_6\}\ \{q_4\}\ \{q_2, q_5\}$

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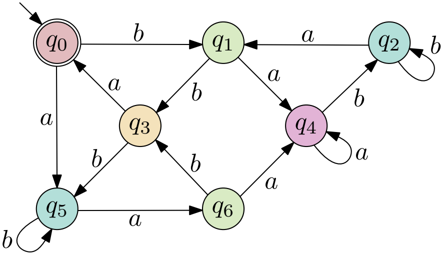

	q_0	q_1	q_2	q_3	q_4	q_5	q_6
q_0							
q_1	ε						
q_2	ε	ba					
q_3	ε	a	a				
q_4	ε	ba	aba	a			
q_5	ε	ba		a	aba		
q_6	ε		ba	a	ba	ba	

$a: \quad \{q_0\}\ \{q_1, q_2, q_3, q_4, q_5, q_6\}$

$b: \quad \{q_0\}\ \{q_3\}\ \{q_1, q_2, q_4, q_5, q_6\}$

$a: \quad \{q_0\}\ \{q_3\}\ \{q_1, q_6\}\ \{q_2, q_4, q_5\}$

$b: \quad \{q_0\}\ \{q_3\}\ \{q_1, q_6\}\ \{q_4\}\ \{q_2, q_5\}$

Minimalautomaten

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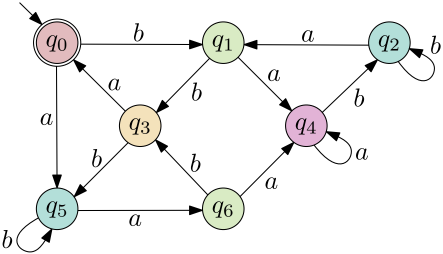

	q_0	q_1	q_2	q_3	q_4	q_5	q_6
q_0							
q_1	ε						
q_2	ε	ba					
q_3	ε	a	a				
q_4	ε	ba	aba	a			
q_5	ε	ba	×	a	aba		
q_6	ε	×	ba	a	ba	ba	

$a: \quad \{q_0\}\ \{q_1, q_2, q_3, q_4, q_5, q_6\}$

$b: \quad \{q_0\}\ \{q_3\}\ \{q_1, q_2, q_4, q_5, q_6\}$

$a: \quad \{q_0\}\ \{q_3\}\ \{q_1, q_6\}\ \{q_2, q_4, q_5\}$

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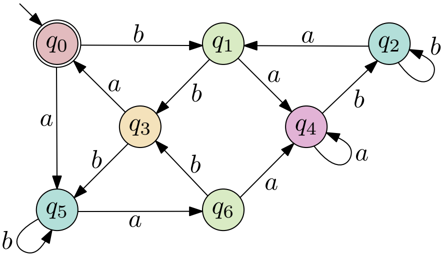
Minimalautomaten



	q_0	q_1	q_2	q_3	q_4	q_5	q_6
q_0							
q_1	ε						
q_2	ε	ba					
q_3	ε	a	a				
q_4	ε	ba	aba	a			
q_5	ε	ba	✗	a	aba		
q_6	ε	✗	ba	a	ba	ba	

$a: \quad \{q_0\}\ \{q_1, q_2, q_3, q_4, q_5, q_6\}$

$b: \quad \{q_0\}\ \{q_3\}\ \{q_1, q_2, q_4, q_5, q_6\}$

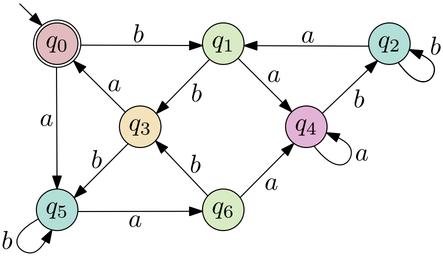
$a: \quad \{q_0\}\ \{q_3\}\ \{q_1, q_6\}\ \{q_2, q_4, q_5\}$

$b: \quad \{q_0\}\ \{q_3\}\ \{q_1, q_6\}\ \{q_4\}\ \{q_2, q_5\}$

$a: \quad \{q_0\}\ \{q_3\}\ \{q_1, q_6\}\ \{q_4\}\ \{q_2, q_5\}$

$\Rightarrow \{q_0\}\ \{q_3\}\ \{q_1, q_6\}\ \{q_4\}\ \{q_2, q_5\}$

Minimalautomaten



$a: \quad \{q_0\}\ \{q_1, q_2, q_3, q_4, q_5, q_6\}$

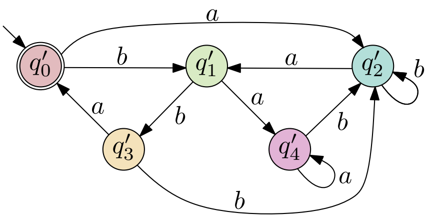
$b: \quad \{q_0\}\ \{q_3\}\ \{q_1, q_2, q_4, q_5, q_6\}$

$a: \quad \{q_0\}\ \{q_3\}\ \{q_1, q_6\}\ \{q_2, q_4, q_5\}$

$b: \quad \{q_0\}\ \{q_3\}\ \{q_1, q_6\}\ \{q_4\}\ \{q_2, q_5\}$

$a: \quad \{q_0\}\ \{q_3\}\ \{q_1, q_6\}\ \{q_4\}\ \{q_2, q_5\}$

$\Rightarrow \{q_0\}\ \{q_3\}\ \{q_1, q_6\}\ \{q_4\}\ \{q_2, q_5\}$



Chomsky-Normalform

Erinnerung: Eine Grammatik $G = (V, \Sigma, P, S)$ ist in **Chomsky-Normalform**, wenn jede Produktion eine der folgenden Formen hat:

- $N \to OP$ mit $N, O, P \in V$
- $N \to t$ mit $N \in V$ und $t \in \Sigma$

Chomsky-Normalform

Erinnerung: Eine Grammatik $G = (V, \Sigma, P, S)$ ist in **Chomsky-Normalform**, wenn jede Produktion eine der folgenden Formen hat:

- $N \rightarrow OP$ mit $N, O, P \in V$
- $N \rightarrow t$ mit $N \in V$ und $t \in \Sigma$

$\Rightarrow \varepsilon \notin L(G)$!

Wir könnten $S \rightarrow \varepsilon$ zulassen, sofern S auf **keiner** rechten Seite vorkommt.

Chomsky-Normalform

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$$G = (V = \{S, A, B, C, D\}, \Sigma = \{a, c, d\}, P, S)$$

$$\begin{aligned} P = \{\, & S \to A \mid aB \mid aC, \\ & A \to B \mid C \mid cAd, \\ & B \to S \mid Ba, \\ & C \to D \mid c, \\ & D \to d \mid dDD \,\} \end{aligned}$$

Chomsky-Normalform

Entfernen von:

1. ε-Produktionen

$$
\begin{aligned}
P = \{\, S &\to A \mid aB \mid aC, \\
A &\to B \mid C \mid cAd, \\
B &\to S \mid Ba, \\
C &\to D \mid c, \\
D &\to d \mid dDD \,\}
\end{aligned}
$$

Chomsky-Normalform

$$P = \{\, S \to A \mid aB \mid aC,$$
$$A \to B \mid C \mid cAd,$$
$$B \to S \mid Ba,$$
$$C \to D \mid c,$$
$$D \to d \mid dDD \,\}$$

Entfernen von:

1. ε-Produktionen ✓

Chomsky-Normalform

$$P = \{\, S \rightarrow A \mid aB \mid aC,\\ A \rightarrow B \mid C \mid cAd,\\ B \rightarrow S \mid Ba,\\ C \rightarrow D \mid c,\\ D \rightarrow d \mid dDD \,\}$$

Entfernen von:

1. ε-Produktionen ✓
2. zyklischen Einheitsproduktionen

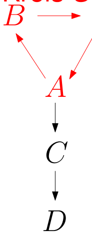
Chomsky-Normalform

$$
\begin{aligned}
P = \{\, & S \rightarrow A \mid aB \mid aC, \\
& A \rightarrow B \mid C \mid cAd, \\
& B \rightarrow S \mid Ba, \\
& C \rightarrow D \mid c, \\
& D \rightarrow d \mid dDD \,\}
\end{aligned}
$$

Entfernen von:

1. ε-Produktionen ✓
2. zyklischen Einheitsproduktionen

Kreis $S \rightarrow A \rightarrow B \rightarrow S$



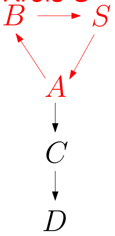
Chomsky-Normalform

$$P = \{\, S \to A \mid aB \mid aC,$$
$$A \to B \mid C \mid cAd,$$
$$B \to S \mid Ba,$$
$$C \to D \mid c,$$
$$D \to d \mid dDD \,\}$$

Entfernen von:

1. ε-Produktionen ✓
2. zyklischen Einheitsproduktionen

Kreis $S \to A \to B \to S$



$$P' = \{\, S \to Sa \mid C \mid cSd \mid aS \mid aC,$$
$$C \to D \mid c,$$
$$D \to d \mid dDD \,\}$$

Chomsky-Normalform

$$P = \{\, S \to Sa \mid C \mid cSd \mid aS \mid aC,\; C \to D \mid c,\; D \to d \mid dDD \,\}$$

Entfernen von:

1. ε-Produktionen ✓
2. zyklischen Einheitsproduktionen ✓
3. nichtzyklischen Einheitsproduktionen

Chomsky-Normalform

$$P = \{ S \to Sa \mid C \mid cSd \mid aS \mid aC,\; C \to D \mid c,\; D \to d \mid dDD \}$$

Entfernen von:

1. ε-Produktionen ✓
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3. nichtzyklischen Einheitsproduktionen

$$P'' = \{ S \to dDD \mid c \mid d \mid Sa \mid cSd \mid aS \mid aC,\; C \to dDD \mid c \mid d,\; D \to dDD \mid d \}$$

Chomsky-Normalform

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$$P = \{\, S \rightarrow dDD \mid c \mid d \mid Sa \mid cSd \mid aS \mid aC,\; C \rightarrow dDD \mid c \mid d,\; D \rightarrow dDD \mid d \,\}$$

Entfernen von:

1. ε-Produktionen ✓
2. zyklischen Einheitsproduktionen ✓
3. nichtzyklischen Einheitsproduktionen ✓
4. langen und gemischten rechten Seiten

Chomsky-Normalform

$$P = \{\, S \to dDD \mid c \mid d \mid Sa \mid cSd \mid aS \mid aC,\; C \to dDD \mid c \mid d,\; D \to dDD \mid d \,\}$$

Entfernen von:

1. ε-Produktionen ✓
2. zyklischen Einheitsproduktionen ✓
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4. langen und gemischten rechten Seiten

$$\begin{aligned} P''' = \{\, & S \to YD \mid c \mid d \mid SX_a \mid X_cZ \mid X_aS \mid X_aC, \\ & C \to YD \mid c \mid d, \\ & D \to YD \mid d, \\ & Y \to X_dD, \\ & Z \to SX_d, \\ & X_a \to a, \quad X_c \to c, \quad X_d \to d \,\} \end{aligned}$$

Chomsky-Normalform

$$P = \{\, S \to A \mid aB \mid aC,$$
$$A \to B \mid C \mid cAd,$$
$$B \to S \mid Ba,$$
$$C \to D \mid c,$$
$$D \to d \mid dDD \,\}$$

Entfernen von:

1. ε-Produktionen ✓
2. zyklischen Einheitsproduktionen ✓
3. nichtzyklischen Einheitsproduktionen ✓
4. langen und gemischten rechten Seiten ✓

$$G^{CNF} = (V' = S, C, D, Y, Z, X_a, X_b, X_c, \Sigma, P''', S)$$

$$P''' = \{\, S \to YD \mid c \mid d \mid SX_a \mid X_cZ \mid X_aS \mid X_aC,$$
$$C \to YD \mid c \mid d,$$
$$D \to YD \mid d,$$
$$Y \to X_dD,$$
$$Z \to SX_d,$$
$$X_a \to a, \quad X_c \to c, \quad X_d \to d \,\}$$