WeGotYouCovered:

The Winning Solver from the PACE 2019 Challenge, Vertex Cover Track

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Mission

- Bridge gap between theory and practice
- Inspire new theoretical developments
- Investigate theoretical algorithms in practice
- Produce accessible implementations & benchmarks
- Encourage dissemination in scientific papers

Previous problems:

- Treewidth, Feedback Vertex Set
- Treewidth, Minimum Fill-In
- Steiner Tree (3 tracks)
**Track 1:** Vertex Cover

**Track 2:** Hybertree Decomposition
- **Track 2a:** Exact
- **Track 2b:** Heuristic

<table>
<thead>
<tr>
<th>Region</th>
<th>Country</th>
<th>Teams</th>
<th>Participants</th>
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<tr>
<td>Europe</td>
<td>Austria</td>
<td>3</td>
<td>3</td>
<td>1a, 2a, 2b</td>
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<td>India</td>
<td>2</td>
<td>5</td>
<td>1a</td>
</tr>
</tbody>
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|                     |               | 10    | 18           | 33             |
Instances

- 200 instances selected out of 9591 from various origins
- 100 public and 100 private instances
- Difficulty rated by time to solve by ILP solver

Selected Instances

- 80 easy [1s;300s)
- 80 medium [300s;1500s)
- 40 hard [1500s;19000s)
Vertex Cover

Given graph $G = (V, E)$, find $S \subseteq V$ s.t.

1. Every edge $e \in E$ is connected to at least one $v \in S$
2. $|S|$ is minimized
Vertex Cover

Given graph $G = (V, E)$, find $S \subseteq V$ s.t.

- Every edge $e \in E$ is connected to at least one $v \in S$
- $|S|$ is minimized

NP hard
Vertex Cover and Complementary Problems

We employ algorithms from all of these problems to tackle vertex cover.
Kernelization

Polynomial Time Kernelization

- Technique from FPT algorithms
- Applies rich set of reduction rules
- Significantly reduces graph size

[|Akiba and Iwata, 2016|]
Reduction Rules

Degree 0

Degree 1

Degree 2

[Akiba and Iwata, 2016]
Reduction Rules

Degree 0

Degree 1

And more...

- Domination
- Unconfined
- Diamond
- LP Relaxation
- Twin
- Funnel
- Desk

Contract into single vertex

[Akiba and Iwata, 2016]
Branch and Reduce

- Reduce graph after each branch
- Additional branching rules to reduce graph size
- Prune search based on lower bounds

[Akiba and Iwata, 2016]
Iterated Local Search

- Originally developed for independent sets
- Perturbation and tabu lists to escape local optima
- Can often find (near-)optimal solutions

[Andrade et al. 2012]
Branch and Bound

- Originally developed for maximum cliques
- Incremental MaxSAT reasoning to prune search
- Combination of static and dynamic vertex ordering

[Li et al. 2017]
Algorithm Overview

ILS + BnR

Kern. + BnB

BnB

time
Instances Solved Over Time

Instances solved

Time $t$ (s)

0  1  10  100  1000

Instances solved

BnB
Kern. + BnB
ILS + BnR
BnR

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BnB
Kern. + BnB
ILS + BnR
BnR

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PACE Results

[Dzulfikar et al. 2019]

76 solved

77 solved

87 solved
Conclusion

Lessons learned / future work
- Heuristics can help branch and bound algorithms
- Algorithm selection is hard!
- What makes an instance hard for one algorithm but easy for another?
- When does kernelization “fail”?

Acknowledgements
- Takuya Akiba and Yoichi Iwata
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- Chu-Min Li, Hua Jiang and Felip Manyà
- Johannes K. Fichte and Markus Hecher

Code: github.com/sebalamm/pace-2019 (algo2.iti.kit.edu/kamis)
References


More Reduction Rules

 reductions

- LP-relaxation

  \[ \text{Minimize} \sum x_v \text{ where } x_u + x_v \geq 1. \]

  If \( x_v = 1 \), then in some MVC.

- Unconfined

  Some MVC exists containing “unconfined” vertices

- Twin

  Generalization of vertex folding

- Diamond, alternative, …