New Solution

Definition (Previous Smaller Values)

For array index $i$ in $A$, let

$$PSV(i) = \arg\max\{k < i : A[k] < A[i]\}$$

be the previous smaller value left of $i$. 
**New Solution**

**Definition (Previous Smaller Values)**

For array index \( i \) in \( A \), let

\[
PSV(i) = \arg\max\{k < i : A[k] < A[i]\}
\]

be the **previous smaller value** left of \( i \).

**Definition (2d-Min-Heap of array \( A \))**

Ordered Tree on nodes \([1, n]\) defined by \( \text{parent}(i) = PSV(i) \).
Lemma

$\text{RMQ}(i, j)$ is given by the child of $LCA(i, j)$ that is on the path to $j$. 
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Represent heap succinctly by DFUDS:

- list degrees of nodes in pre-order:
  - node of out-degree $k \Rightarrow \binom{k}{1}$
  - space $2n$ bits
  - array-index $i$ corresponds to $i$'th ‘)’

There is a preprocessing scheme of optimal size $2n + o(n)$ bits for $O(1)$-range minimum queries. Workspace is also $O(n)$ bits.
Represent heap **succinctly** by DFUDS:

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Theorem

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Represent heap succinctly by DFUDS:
- list degrees of nodes in pre-order:
- node of out-degree \( k \) \( \Rightarrow \binom{k}{k} \)
  \( \Rightarrow \) space \( 2n \) bits
  \( \Rightarrow \) array-index \( i \) corresponds to \( i^{th} \)'')
- \( O\left(\frac{n \log \log n}{\log n}\right) \)-bit index for simulating \( O(1) \)-LCAs (technical!)
- DFUDS can be constructed “in-place”
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**Theorem**

*There is a preprocessing scheme of optimal size \( 2n + o(n) \) bits for \( O(1) \)-range minimum queries. Workspace is also \( O(n) \) bits.*
More Functionality: PSV

**Definition (2d-Min-Heap)**

Ordered Tree defined by \( \text{parent}(i) = \text{PSV}(i) \).

\[
\text{PSV}(i) = \max\{ k < i : H[k] < H[i] \}
\]

⇒ PSV simple (move to parent in \( O(1) \) time!)
More Functionality: PSV

**Definition (2d-Min-Heap)**

Ordered Tree defined by $parent(i) = PSV(i)$.

- $PSV(i) = \max\{k < i : H[k] < H[i]\}$
- $NSV(i) = \min\{k > i : H[k] < H[i]\}$
More Functionality: PSV

Definition (2d-Min-Heap)
Ordered Tree defined by \( \text{parent}(i) = \text{PSV}(i) \).

- \( \text{PSV}(i) = \max\{k < i : H[k] < H[i]\} \)
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Can we also do NSV???
More Functionality: NSV

\[
\begin{align*}
\text{NSV} (i_3) &= i_k + |T_{i_k}| \\
\end{align*}
\]
More Functionality: NSV

1. Find leftmost $<$-sibling to the right
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2. If it does not exist...
3. \( \ldots NSV(i_3) = i_k + |T_{i_k}| \)
More Functionality: NSV

- Distinguish $\equiv$- and $<$-siblings?
- Mark $<$-children in additional bit-vector
- Bit-tricks for $\mathcal{O}(1)$-computations

Theorem (Extended 2d-Min-Heap)

$3n + o(n)$ bits suffice to support RMQ, PSV and NSVs in $\mathcal{O}(1)$ time.
More Functionality: NSV

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- Mark $<$-children in additional bit-vector
- Bit-tricks for $\mathcal{O}(1)$-computations

**Theorem (Extended 2d-Min-Heap)**

$3n + o(n)$ bits suffice to support RMQ, PSV and NSVs in $\mathcal{O}(1)$ time.

- Not necessarily optimal...
- $\ldots \leq 2.54 \ldots n$ possible (Schröder Tree!)