Text Indexing: Lecture 10

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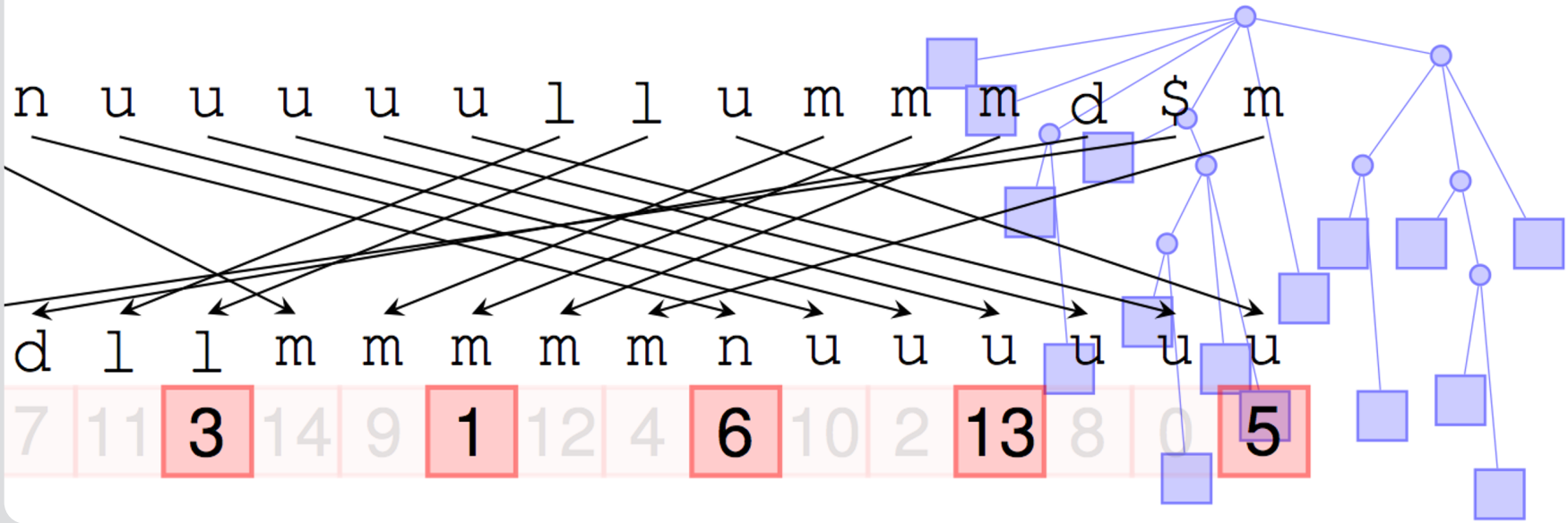
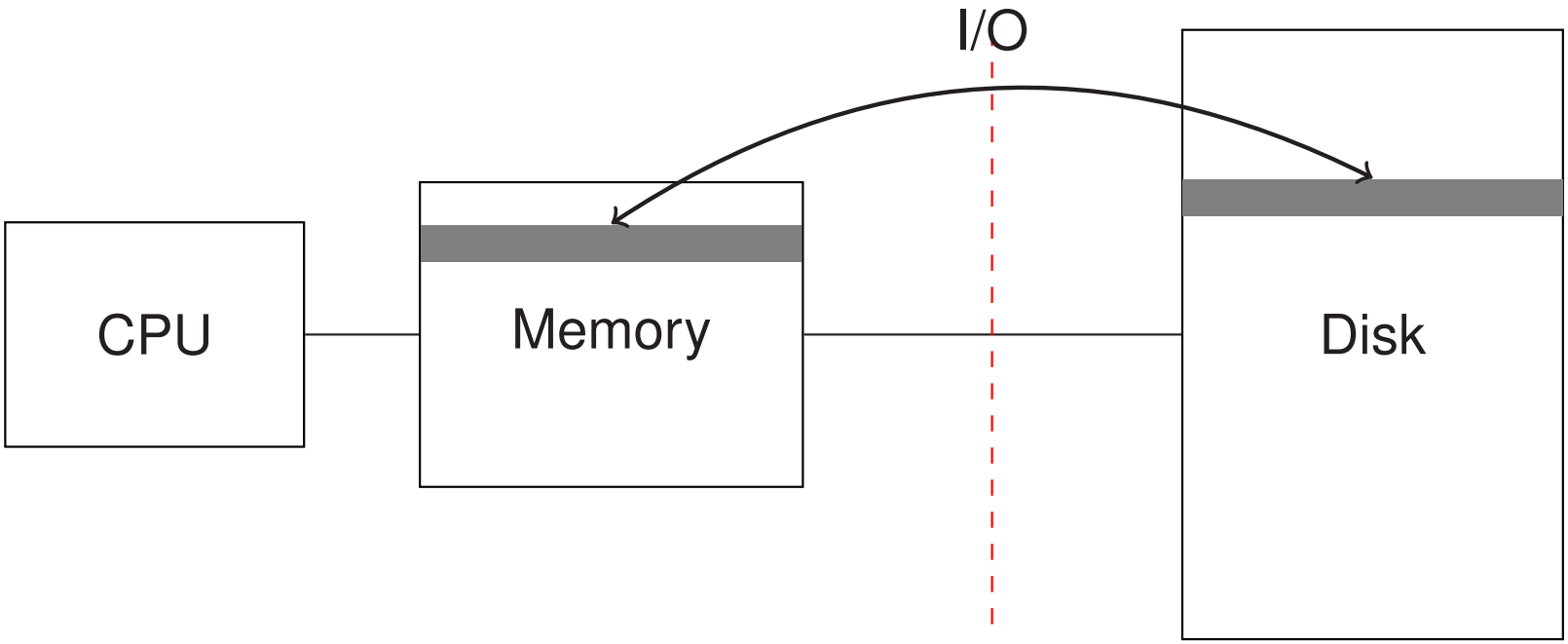


External Memory Model



Parameters:

B = # of items per disk block

M = # of items that fit into main memory

Cost model: Only I/Os (block moves between main memory and disk) are counted.

Exercise

Karlsruhe Institute of Technology

Question

What is the I/O complexity for count queries using

- suffix trees?
- suffix arrays?
- FM-indexes?

The String-B Tree

Problem

Given a set $S = \{s_0, \ldots, s_{N-1}\}$ of N strings of total length n. A *prefix count query* asks for the subset of strings which starts with a given pattern P. Devise a data structure that answers prefix count queries efficiently in the external memory model.

Solution (Ferragina & Grossi, SODA 1996)

String-B Trees (SBT) is a combination of B-trees and Patrica tries. It allows prefix count queries in $\mathcal{O}(\frac{|P|}{B} + \log_B N)$.

- Keys in the SBT are pointers to the text (stored in external memory)
- Keys are stored at leaves; inner nodes store copies of a subset of keys
- Key ordering is determined by lexicographical ordering of the corresponding strings

The String-B Tree

- Each SBT node v is stored in a disk block and contains an ordered string set $S_v \subset S$
- $b \leq S_v \leq 2b$ for $b \in \Theta(B)$
- Denote the leftmost (rightmost) string in S_v by $L(v)$ resp. $R(v)$
- Construction:
  - Partition S into groups of b strings (last group may have less than b strings)
  - Each group is mapped to a leaf such that a left-to-right scan of the SBT gives the strings in lex. order.
  - The longest common prefix $lcp(S_j, S_{j+1})$ is associated with each pair.

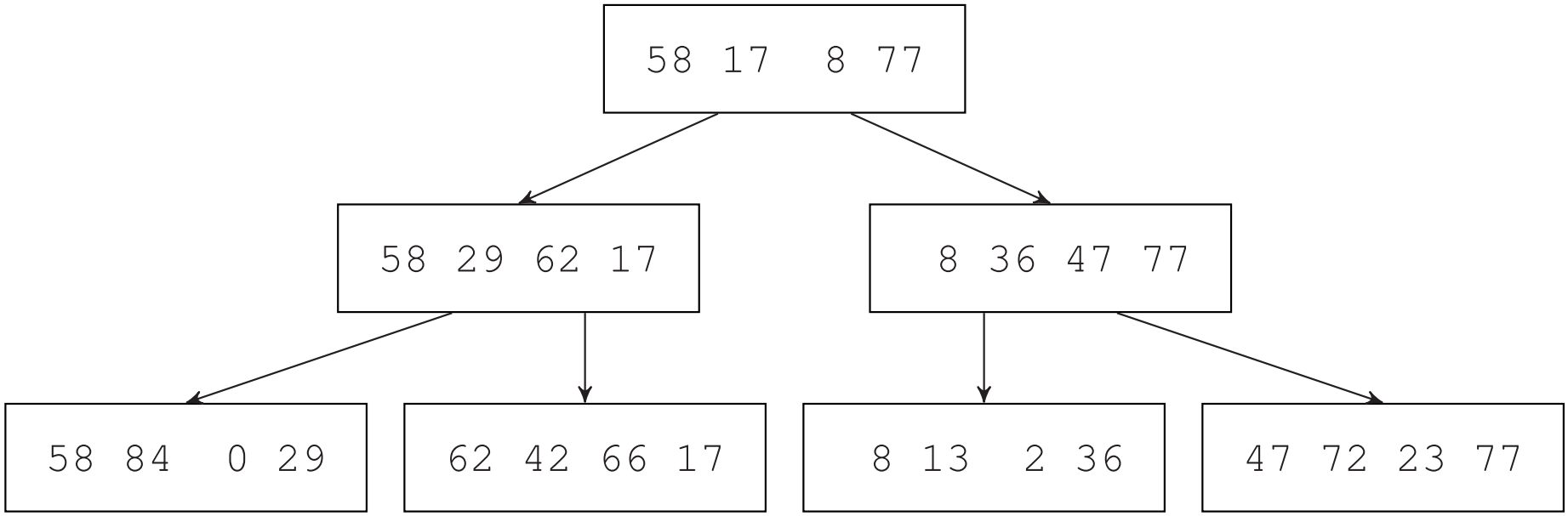
  - Each internal node v has $d(v)$ children $u_0, \ldots, u_{d(v)-1}$, with $b/2 \leq d(v) \leq b$ (root node may have 2 to b children).
  - Set S_v is formed by copying the leftmost and rightmost strings of $v's$ children. I.e. $S_v = \{L(u_0), R(u_0), \ldots, L(u_{d(v)-1}), R(u_{d(v)-1})\}$.

Example

```
                1   1     2     2       3     4
0 2     8       3   7     3     9       6     2
i think that the ruddy widow really wants ripe

4           5   6   6     7     7       8
7           8   2   6     2     7       4
watermelon and red roses when winter arrives
```

Example

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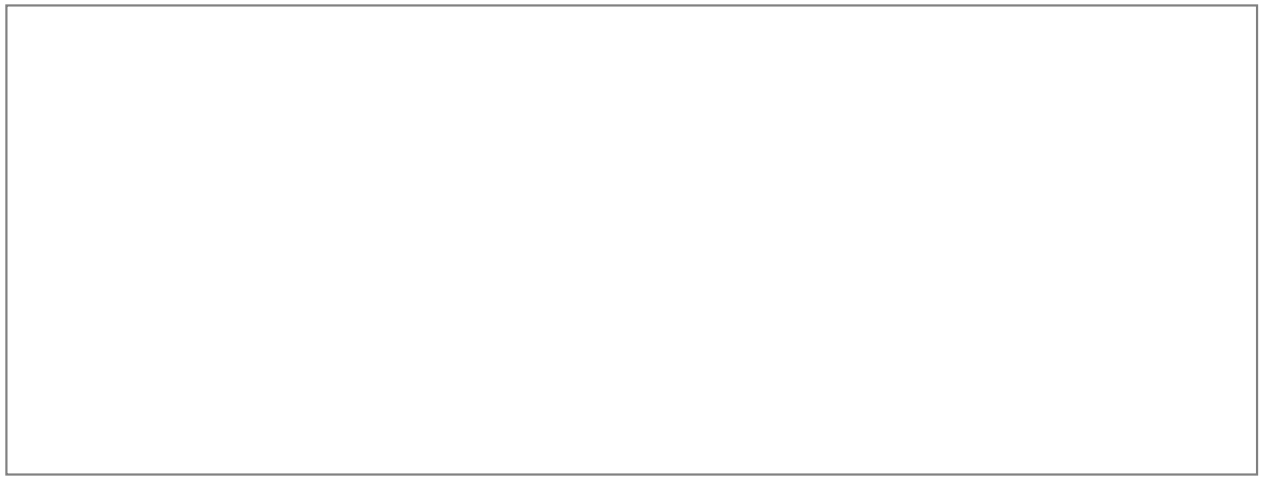
Search in the SBT

- String set S_v is represented as a blind trie
- Build a patricia trie over the binary strings of S_v
- Example:



- blind trie is stored in a succinct tree representation
- blind trie matching does not require further IOs
- follow blind search until reaching a leaf
- check if P occurs at the pointer returned by the blind search in the leaf

Time and space

Time

- Count queries: $\mathcal{O}(|P|/B + \log_B N)$ (assuming that subtree sizes are stored in nodes)
- Locate queries: $\mathcal{O}((|P| + Z)/B + \log_B N)$, where Z is the number of strings in S which are prefixed by P

Space

$\mathcal{O}(N/B)$ leaves. Summing up all inner nodes results in $\mathcal{O}(N/B)$ nodes in the SBT. Each node fits into a disk block (which stores B items).

Bibliography

Paolo Ferragina and Roberto Grossi.
Fast string searching in secondary storage: Theoretical developments and experimental results.
In *Proc. SODA*, pages 373–382, 1996.

Juha Kärkkäinen and S. Srinivasa Rao.
Full-Text Indexes in External Memory, volume 2625 of *Lecture Notes in Computer Science*, chapter Chapter 7, pages 149–170.
Springer, Berlin, Germany, 2003.
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