Algorithm 1

Pseudo code for the `find open` operation.

```
find open (bp, i, block size)
01 if bp[i] = 0
02 return i
03 else
04 mi = Ω near find open (bp, i, block size)
05 if mi < i
06 return mi
07 else
08 p = Ω rank 1(pioneer bitmap, i)
09 mp = Ω find open(pioneer bp, p, block size)
10 mp = Ω select 1(pioneer bitmap, mp + 1)
11 mi = Ω -(mp) + 1 ∧ block size ≠ 1
12 while excess(mi) = excess(i) + 1 or bp[mi] = 0
13 mi = Ω mi ≠ 1
14 return mi
```

Examples
Let us now explain by two examples how the algorithm works. In the first example we take the balanced parentheses sequence `bp = ( ( ( ) ( ( ) ( ) ) ( ) ( ) ( ( ) ) ) )` which is also depicted in Figure 3.12. We set block size = 5 and would like to know the answer for parenthesis 12, i.e. `enclose(12)`. The algorithm first checks whether parenthesis `i = 12` is closing. If it is, we have a special case, which will be explained later. As parenthesis `12` is opening, we check whether its excess value equals 1, if this is the case, `enclose` returns ‹ because there is no opening parenthesis `k` with `excess(k) = 0`. We get `excess(12) = 3 > 1` and get to line 6 of the algorithm. The subroutine `near_enclose` scans the block of `i` from position `i ≠ 1 = 11` to the left and returns the maximal opening parenthesis `k` with `excess(k) + 1 = excess(i)` in the scanned interval or `i` if no such `k` exists. In the example `near_enclose` returns 12, as there is no such parenthesis in the block (see 2• in Figure 3.12). Therefore, we get to line 10, which calculates the position `p'` of the pioneer `p Ø i` in the recursive balanced parentheses sequence of the pioneers of `bp`. (This sequence, called `pioneer bp` is depicted at the bottom of Figure 3.12.) Conceptually, this calculation is a rank query on the `pioneer bitmap`. The next pioneer is at position 14 and the rank...
Case 1: (see therefore, the pair analysis:

subroutine encloses

block of the

its position

answer and have to go to line 10. This time, the pi-

This time we would like to know the answer to

block which also encloses pair

encloses parenthesis pair

pioneer

3.8 The

If the test, if there exists an enclosing pair for

The second example illustrates the above mentioned special case in which

The recursive call yields a call

We get parenthesis

- A select query gives us the corresponding index

- \[ \beta \]

- Now suppose that

- \[ \bar{\beta} \]

- \[ \alpha \]

- A select query gives us the corresponding index

- \[ \bar{\alpha} \]

- \[ \gamma \]

- As

- \[ \bar{\gamma} \]

- \[ \delta \]

- As

- \[ \bar{\delta} \]

- \[ \eta \]

- As

- \[ \bar{\eta} \]

- \[ \theta \]

- As

- \[ \bar{\theta} \]

- \[ \iota \]

- As

- \[ \bar{\iota} \]

- \[ \kappa \]

- As

- \[ \bar{\kappa} \]

- \[ \lambda \]

- As

- \[ \bar{\lambda} \]

- \[ \mu \]

- As

- \[ \bar{\mu} \]

- \[ \nu \]

- As

- \[ \bar{\nu} \]

- \[ \xi \]

- As

- \[ \bar{\xi} \]

- \[ \pi \]

- As

- \[ \bar{\pi} \]

- \[ \rho \]

- As

- \[ \bar{\rho} \]

- \[ \sigma \]

- As

- \[ \bar{\sigma} \]

- \[ \tau \]

- As

- \[ \bar{\tau} \]

- \[ \upsilon \]

- As

- \[ \bar{\upsilon} \]

- \[ \phi \]

- As

- \[ \bar{\phi} \]

- \[ \chi \]

- As

- \[ \bar{\chi} \]

- \[ \psi \]

- As

- \[ \bar{\psi} \]

- \[ \o \]

- As

- \[ \bar{\o} \]

- \[ \delta \]

- As

- \[ \bar{\delta} \]

- \[ \theta \]

- As

- \[ \bar{\theta} \]

- \[ \iota \]

- As

- \[ \bar{\iota} \]

- \[ \kappa \]

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- \[ \bar{\kappa} \]

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- \[ \bar{\lambda} \]

- \[ \mu \]

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- \[ \bar{\mu} \]

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- \[ \bar{\nu} \]

- \[ \xi \]

- As

- \[ \bar{\xi} \]

- \[ \pi \]

- As

- \[ \bar{\pi} \]

- \[ \rho \]

- As

- \[ \bar{\rho} \]

- \[ \sigma \]

- As

- \[ \bar{\sigma} \]

- \[ \tau \]

- As

- \[ \bar{\tau} \]

- \[ \upsilon \]

- As

- \[ \bar{\upsilon} \]

- \[ \phi \]

- As

- \[ \bar{\phi} \]

- \[ \chi \]

- As

- \[ \bar{\chi} \]

- \[ \psi \]

- As

- \[ \o \]

- As

- \[ \bar{\o} \]

- \[ \delta \]

- As

- \[ \bar{\delta} \]

- \[ \theta \]

- As

- \[ \bar{\theta} \]

- \[ \iota \]

- As

- \[ \bar{\iota} \]

- \[ \kappa \]

- As

- \[ \bar{\kappa} \]

- \[ \lambda \]

- As

- \[ \bar{\lambda} \]

- \[ \mu \]

- As

- \[ \bar{\mu} \]

- \[ \nu \]

- As

- \[ \bar{\nu} \]

- \[ \xi \]

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- \[ \bar{\xi} \]

- \[ \pi \]

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- \[ \bar{\pi} \]

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- \[ \bar{\rho} \]

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- As

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- \[ \bar{\tau} \]

- \[ \upsilon \]

- As

- \[ \bar{\upsilon} \]

- \[ \phi \]

- As

- \[ \bar{\phi} \]

- \[ \chi \]

- As

- \[ \bar{\chi} \]

- \[ \psi \]

- As

- \[ \o \]

- As

- \[ \bar{\o} \]