Automated Planning and Scheduling
Lecture 4: Heuristics for State Space Search
Tomáš Balyo, Dominik Schreiber | November 6, 2019
Outline

- Motivation for Heuristics in Search algorithms
- A (non-exhaustive) overview on common heuristics for planning
- Some remarks on designing heuristics in practice
Heuristics: What we already know (1)

- Path finding in a uniform 2D grid
- Dijkstra’s algorithm: Uninformed search, explores space breadth-first

Better: Use a heuristic to guide the search (e.g. best-first, A∗)

E.g. Euclidean distance to the goal as a lower bound of how much is left

Traversed subspace almost "one-dimensional", depending on search strategy and heuristic

Imagine the difference in an n-dimensional grid!
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- Imagine the difference in an n-dimensional grid!
Planning by State-Space Search is basically the same problem
- High-dimensional search space, action sequences are possible paths

- In A* search, nodes are prioritized by their value
  \[ f(n) = c(n) + h(n) \]
  \( c(n) \): Cost to get to this node
  \( h(n) \): Heuristic: Lower bound for the remaining cost to the goal

- What does this mean in state space?
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Applying “the A* idea” to domain-independent planning
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Heuristics: What we already know (2)

- Planning by State-Space Search is basically **the same problem**
  - High-dimensional search space, action sequences are possible paths
- Applying “the A* idea” to **domain-independent planning**
  - In A*, nodes are prioritized by their value $f(n) = c(n) + h(n)$
    - $c(n)$: Cost to get to this node
    - $h(n)$: **Heuristic**: Lower bound for the remaining cost to the goal
  - $c(n) := \text{sum of action cost so far}$
    - $= \text{sum of actions, if cost per action is constant}$
  - $h(n) := \text{euclidean distance to goal}$
    - What does this mean in state space?
A first heuristic for planning (1)

- Imagine state space as the corners of a $|\mathcal{P}|$-dimensional hypercube
  - Every atom corresponds to one edge, its possible values true and false are the adjacent corners

How do we measure Euclidean distances in $|\mathcal{P}|$ dimensions?

$$d(p, q) = \sqrt{\sum_{i=1}^{|\mathcal{P}|} (p_i - q_i)^2}$$

In our discrete sub-space: for two states $p$ and $q$,

$$(p_i - q_i)^2 = 1 \text{ if they differ in the } i\text{-th atom, }$$

$$(p_i - q_i)^2 = 0, \text{ else}$$

How do we measure the distance from a state $s$ to our goal $g$?

Only consider the sub-space involving the atoms in the goal

Leads to

$$d(s, g) := \sqrt{\sum_{a \in g} 1 - \left[ a \in s \right]}$$
A first heuristic for planning (1)

- Imagine state space as the corners of a $|\mathcal{P}|$-dimensional hypercube
- Every atom corresponds to one edge, its possible values $\text{true}$ and $\text{false}$ are the adjacent corners
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- How do we measure the distance from a state $s$ to our goal $g$?
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How do we measure euclidean distances in $|\mathcal{P}|$ dimensions?

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In our discrete sub-space: for two states $p$ and $q$,
\[ (p_i - q_i)^2 = 1 \text{ if they differ in the } i\text{-th atom,} \]
\[ (p_i - q_i)^2 = 0, \text{ else} \]

How do we measure the distance from a state $s$ to our goal $g$?

- Only consider the sub-space involving the atoms in the goal
- Leads to $d(s, g) := \sqrt{\sum_{a \in g} 1 - [a \in s]}$
A first heuristic for planning (2)

- Truck example
  - Goals: \text{at}(\text{package1, loc3}), \text{at}(\text{package2, loc3}), \text{at}(\text{package3, loc3})
  - Initial state: None of above atoms are true
- Interpret initial state as point $s_0 = (0, 0, 0)$ and goal as $g = (1, 1, 1)$, one dimension per atom
- Initially, $d(s, g) = d(s_0, g)$
Truck example

- Goals: `at(package1, loc3)`, `at(package2, loc3)`, `at(package3, loc3)`
- Initial state: None of above atoms are true
- Interpret initial state as point $s_0 = (0,0,0)$ and goal as $g = (1,1,1)$, one dimension per atom
- Initially, $d(s, g) = d(s_0, g) = \sqrt{3}$
A first heuristic for planning (2)

- Truck example
  - Goals: at(package1, loc3), at(package2, loc3), at(package3, loc3)
  - Initial state: None of above atoms are true
- Interpret initial state as point $s_0 = (0, 0, 0)$ and goal as $g = (1, 1, 1)$, one dimension per atom
- Assume we bring package1 to loc3
A first heuristic for planning (2)

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  - Goals: $\text{at}(\text{package1}, \text{loc3})$, $\text{at}(\text{package2}, \text{loc3})$, $\text{at}(\text{package3}, \text{loc3})$
  - Initial state: None of above atoms are true
- Interpret initial state as point $s_0 = (0, 0, 0)$ and goal as $g = (1, 1, 1)$, one dimension per atom
- Assume we bring package1 to loc3
  $\Rightarrow d(s, g) = \sqrt{2}$
A first heuristic for planning (2)

- Truck example
  - Goals: \text{at}(\text{package1}, \text{loc3}), \text{at}(\text{package2}, \text{loc3}), \text{at}(\text{package3}, \text{loc3})
  - Initial state: None of above atoms are true
- Interpret initial state as point $s_0 = (0, 0, 0)$ and goal as $g = (1, 1, 1)$, one dimension per atom
- Assume we also bring \text{package2} to \text{loc3}
Truck example
- Goals: \( \text{at}(\text{package}1, \text{loc}3), \text{at}(\text{package}2, \text{loc}3), \text{at}(\text{package}3, \text{loc}3) \)
- Initial state: None of above atoms are true
- Interpret initial state as point \( s_0 = (0, 0, 0) \) and goal as \( g = (1, 1, 1) \), one dimension per atom
- Assume we also bring \( \text{package}2 \) to \( \text{loc}3 \)
  \[ d(s, g) = 1 \]
A first heuristic for planning (2)

- Truck example
  - Goals: at(package1, loc3), at(package2, loc3), at(package3, loc3)
  - Initial state: None of above atoms are true
- Interpret initial state as point $s_0 = (0, 0, 0)$ and goal as $g = (1, 1, 1)$, one dimension per atom
- When all packages are where they should be: $d(s, g) = 0$
Admissible heuristics (1)

Admissibility.

A heuristic $h(s)$ regarding a goal $g$ is admissible iff $\forall s : h(s) \leq h^*(s)$, where $h^*(s)$ is the actual remaining distance to the goal $g$.

- What does an admissible heuristic imply if using $A^*$?
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- What does an admissible heuristic imply if using A*?  
  $\Rightarrow$ Optimal plan will be found
- Optimality in basic STRIPS planning: Shortest plan
- Optimality in planning with (non-uniform) action costs:  
  Plan with lowest total cost
  - Much more complex to find
- In the following, we mostly stay with uniform action cost
Admissible heuristics (2)

Admissibility.

A heuristic $h(s)$ regarding a goal $g$ is \textit{admissible} iff $\forall s : h(s) \leq h^*(s)$, where $h^*(s)$ is the actual remaining distance to the goal $g$.

- Our heuristic: $h_{euc}(s) := d(s, g) = \sqrt{\sum_{a \in g} 1 - [a \in s]}$

- Is this heuristic \textit{admissible} (for uniform action costs)?
Admissible heuristics (2)

**Admissibility.**

A heuristic \( h(s) \) regarding a goal \( g \) is *admissible* iff \( \forall s : h(s) \leq h^*(s) \), where \( h^*(s) \) is the actual remaining distance to the goal \( g \).

- Our heuristic: \( h_{euc}(s) := d(s, g) = \sqrt{\sum_{a \in g} 1 - [a \in s]} \)
- Is this heuristic *admissible* (for uniform action costs)? Only if no action satisfies more than one goal atom.
- Discussion: Is the heuristic *useful*?
Admissible heuristics (2)

Admissibility.

A heuristic $h(s)$ regarding a goal $g$ is **admissible** iff $\forall s : h(s) \leq h^*(s)$, where $h^*(s)$ is the actual remaining distance to the goal $g$.

- Our heuristic: $h_{euc}(s) := d(s, g) = \sqrt{\sum_{a \in g} 1 - [a \in s]}$

- Is this heuristic **admissible** (for uniform action costs)?
  
  Only if no action satisfies more than one goal atom.

- Discussion: Is the heuristic **useful**?
  
  - Can be useful if problem has many goals, each of which only take a single action
  - Mostly useless when many actions are required to produce a goal
  - Worst Case: Large planning problems with a single goal; heuristic degenerates to $h_{euc}(s) = 1$ for almost all $s$
Good heuristics wanted

- Any ideas for better heuristics?
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  - Finer grained metric than “amount of satisfied goals”
  - Acknowledge non-goal atoms in the state as well
  - Consider applicable actions and which changes they would bring
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- Attributes of quality for a planning heuristic
  - Admissibility (although not always required)
  - Significance: should convey useful information of appropriate resolution
  - Accuracy: as close as possible to $h^*(s)$
  - Computability: Easier to compute than the original problem!
    Allowed complexities: Linear? Polynomial? Even NP-complete?
The central paradigm: Relaxation

- We cannot hope to find the true goal distance from all $s$ for our heuristic
  - Computing it already implies finding a path (= a plan)!
  - Even worse when dealing with action costs
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- We cannot hope to find the true goal distance from all $s$ for our heuristic
  - Computing it already implies finding a path (= a plan)!
  - Even worse when dealing with action costs
- Instead, fall back to a simplified problem (relaxation)
  - Easier to compute and analyze
  - Provides at least certain bounds for the original problem’s properties
- New objective: Find such a relaxation for planning problems
Delete-relaxation [GNT16]

Let $\pi = (P, A, s_I, G)$ a planning problem. Then $\pi_r := (P, A_r, s_I, G)$ is called the delete-relaxation of $\pi$, whereas $A_r = \{a_r | a \in A\}$, with $\text{pre}(a_r) = \text{pre}^+(a)$ and $\text{eff}(a_r) = \text{eff}^+(a)$.

Intuitively, $\pi_r$ only considers positive atoms at each state. Negative preconditions and negative (“delete”) effects of actions are neglected. Resulting problem corresponds to $\text{PLANSAT}$ (see lecture 2).

Important property: All valid action sequences in $\pi$ from some state are also valid in $\pi_r$ from the same state. Problem never gets harder in any way.
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  - Resulting problem corresponds to $\text{PLANSAT}^+_\dagger$ (see lecture 2)
- Important property: All valid action sequences in $\pi$ from some state $s$ are also valid in $\pi_r$ from $s$
  - Problem never gets harder in any way
Making use of delete-relaxed problems

What can we do with $\pi_r$?

- All effects are positive: Applying actions leads to increasing set of atoms until some fixpoint (or until all goals are reached)
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- Construct **relaxed planning graph**
  - Planning graph: Alternation of states and applicable actions
  - Relaxation: Collapse possible states, execute **all applicable actions at once**
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```
truck-at(A)
at(p1,A)
at(p2,B)
{pick-up(p1), drive(B)}

truck-at(A)
truck-at(B)
at(p1,A)
at(p2,B)
in-truck(p1)

{pick-up(p1), pick-up(p2), drive(A), drive(B), drive(C), drop(p1)}

truck-at(A)
truck-at(B)
truck-at(C)
at(p1,A)
at(p1,B)
at(p2,B)
in-truck(p1)
in-truck(p2)
{...}
```
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Delete-relaxation: A first heuristic

Let’s assemble a heuristic:

- Given a state $s$, begin a relaxed planning graph starting at $s$
- Build graph layer for layer until all (positive) goals are reached
- Let $h_d^+(s) :=$ the required depth, i.e. amount of needed iterations
  - What if goals are not reached in the graph?

Discussion: Admissible? Useful? Easy to compute?

- Admissible: only for action cost $\geq 1$ (see exercises)
- Useful if relaxed problem has long critical path leading to the goals
- Less useful if relaxed planning graph is very shallow
- E.g. if almost all action preconditions are negative

Computational complexity: Easy (see complexity of PLANSAT)
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- Computational complexity: Easy (see complexity of \( \text{PLANSAT}^+ \))

Intermission: \( h^+_d(s) \) in Aquaplanning
The Fast-Forward Heuristic (1)

- $h_d^+$ is inaccurate / has low resolution: few information on actual amount of required actions

Build relaxed planning graph $G$ from $s$ until relaxed goal $g$ is satisfied ($\ast$) Extract an actual (relaxed) solution plan $p$ Return cost of $p$ Important idea for ($\ast$): If one can execute multiple actions $A$ at the same state, the ordering of actions does not matter

Reasoning:
1. All preconditions and effects are positive
2. All actions are already applicable
   $\implies$ None of the actions will disable another one
The Fast-Forward Heuristic (1)

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- More accurate: Fast-Forward heuristic $h^{FF}(s)$ [HN01]
  - Build relaxed planning graph $G$ from $s$ until relaxed goal $g^+$ is satisfied
  - (*) Extract an actual (relaxed) solution plan $p$
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- Important idea for (*): If one can execute multiple actions $A$ at the same state, the ordering of actions does not matter
  - Reasoning:
    1. All preconditions and effects are positive
    2. All actions are already applicable
   $\Rightarrow$ None of the actions will disable another one
The Fast-Forward Heuristic (2)

How to compute the relaxed plan?

1. \( \hat{s} := \text{1st state in } G \text{ which satisfies } g^+; \) \( \hat{g} := g^+; \) \( p := \langle \rangle; \)
2. While \( \hat{s} \neq s: \)
3. \( \hat{s} := \text{predecessor of } \hat{s} \text{ in } G; \)
4. Choose set of actions \( A \) applicable in \( \hat{s} \) such that \( \gamma^+(\hat{s}, A) \) satisfies \( \hat{g}; \)
5. \( p := A \circ p; \)
6. \( \hat{g} := (\hat{g} \setminus \text{eff}^+(A)) \cup \text{pre}^+(A); \)
7. Output \( p. \)

\[
\begin{align*}
\text{truck-at}(A) & \quad \text{at}(p1,A) \\
& \quad \text{at}(p2,B) \\
\text{pick-up}(p1) & \quad \text{drive}(B)
\end{align*}
\]
The Fast-Forward Heuristic (2)

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2. While $\hat{s} \neq s$:
3. $\hat{s} := \text{predecessor of } \hat{s} \text{ in } G$;
4. Choose set of actions $A$ applicable in $\hat{s}$ such that $\gamma^+(\hat{s}, A)$ satisfies $\hat{g}$;
5. $p := A \circ p$; (Apply positive effects only)
6. $\hat{g} := (\hat{g} \setminus \text{eff}^+(A)) \cup \text{pre}^+(A)$;
The Fast-Forward Heuristic (2)

How to compute the relaxed plan?

1. \( \hat{s} := 1^{st} \) state in \( G \) which satisfies \( g^+ \); \( \hat{g} := g^+ \); \( p := \langle \rangle \);
2. While \( \hat{s} \neq s \):
3. \( \hat{s} := \) predecessor of \( \hat{s} \) in \( G \);
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\[
\begin{align*}
\text{truck-at}(A) \quad & \text{pick-up}(p1) \\
\text{at}(p1,A) \quad & \text{drive}(B) \\
\text{at}(p2,B) \quad & \\
\text{pick-up}(p1) \quad & \\
\text{pick-up}(p2) \quad & \\
\text{drive}(A) \quad & \\
\text{drive}(B) \quad & \\
\text{drive}(C) \quad & \\
\text{in-truck}(p1) \quad & \\
\text{drop}(p1) \quad & \\
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\text{at}(p2,B) \quad & \text{drive}(C) \\
\text{in-truck}(p1) \quad & \text{drop}(p1) \\
\text{in-truck}(p2) \quad & \text{drive}(B) \\
\text{at}(p2,{A,B,C}) \quad & \text{drive}(C) \\
\text{in-truck}(p1) \quad & \text{drop}(p2) \\
\text{in-truck}(p2) \quad &
\end{align*}
\]

⇒ \( p = \langle \text{drive}(B), \text{pick-up}(p1), \text{pick-up}(p2), \text{drive}(C), \text{drop}(p1), \text{drop}(p2) \rangle \)
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\[ \Rightarrow p = \langle \text{drive}(B), \text{pick-up}(p1), \text{pick-up}(p2), \text{drive}(C), \text{drop}(p1), \text{drop}(p2) \rangle \]
Properties of $h^{FF}$

When is $h^{FF}$ admissible?

- $h^{FF}$ admissible $\iff \forall s : h^{FF}(s) \leq h^*(s)$
  $\iff$ cost of relaxed extracted $p \leq$ cost of exact plan $p^*$
  $\iff$ $p$ never “overestimates” $p^*$
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- Uniform action costs: always pick minimum set of actions $A$
  - Equivalent to set cover problem (NP-complete)

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  - Equivalent to set cover problem (NP-complete)
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Consequence: $h^{FF}(s)$ can’t be both admissible and easy to compute

- Essential cause: Finding an optimal plan is difficult even in delete-relaxed problems (Does this look familiar?)
Original implementation of $h^{FF}$ in the Fast-Forward planner [HN01]:

- Works with general action costs
- Choice of actions is approximately to be near-optimal
FF: Concluding remarks

Original implementation of $h^{FF}$ in the Fast-Forward planner [HN01]:

- Works with general action costs
- Choice of actions is approximated to be near-optimal
- Heuristic search complemented by many additional techniques contributing to the success of FF
  - Enforced hill climbing (see last lecture)
  - Pruning of unpromising nodes:
    Only apply actions considered helpful regarding the relaxed solution
- Much more
Delete-relaxed Landmark heuristics

Crude idea of Landmarks:

- Try to find common points which all valid plans share
- How many such points are still missing? ⇒ Use for heuristic
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Disjunctive Action Landmarks [HR15]

A disjunctive action landmark is a set of actions $A$ such that each delete-relaxed plan contains some $a \in A$. 
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Landmarks in Trucking example:
- $L_1 = \{\text{pickup}(1), \text{pickup}(2)\}$
- $L_2 = \{\text{move}(B)\}, L_3 = \{\text{move}(C)\}$
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- $L_2 = \{ \text{move}(B) \}$, $L_3 = \{ \text{move}(C) \}$
- Assume two alternative routes $B_1$ and $B_2$ from $B$ to $C$:
  - $L_4 = \{ \text{move}(B_1), \text{move}(B_2) \}$
Computing Landmarks

- Construct a Justification Graph [HR15]
  - Vertices: Single atoms
  - Directed edges for each action $a$: some precondition of $a$ to each of its effects (needs some precondition choice strategy)

Modified Trucking example (1 package, road $B \rightarrow \{B_1, B_2\} \rightarrow C$):
- Each graph cut (separating start from goal) forms a Landmark!
- At least one of cut actions required to get to goal
- Delete-relaxation, choice of preconditions never increase cut size
- Open question: Which preconditions to choose?
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  - `move(B, B_1)`
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  - `move(B_1, C)`
  - `move(B_2, C)`
  - `drop(p1)`
  - `at(p1, C)`

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Using Landmarks

- Suppose multiple landmarks \( L = \{ L_1, L_2, \ldots, L_k \} \) have been found from state \( s \) to \( g \)
- How to exploit this knowledge for heuristic value \( h_L(s) \)?
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- Suppose multiple landmarks \( L = \{L_1, L_2, \ldots, L_k\} \) have been found from state \( s \) to \( g \).
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  - \( h_L(s) := |L| \), more general: \( h_L(s) := \sum_{L_i \in L} \left( \sum_{a \in L_i} \text{cost}(a) \right) \)
Suppose multiple landmarks $L = \{L_1, L_2, \ldots, L_k\}$ have been found from state $s$ to $g$.

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\( h_L(s) \) can be admissible;
again, trade-off between accuracy and computational effort
Relaxation by abstraction (1)

- Another relaxation: Abstract state space of some atom pattern $X$
  - Example: Consider subspace of pattern $X := \{at(p_1, \cdot), at(p_2, \cdot)\}$
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Plan search in abstracted state space can be much easier
⇒ Exploit this for a heuristic
Relaxation by abstraction (2)

- Construction of abstraction-relaxed problem: easy
  - Remove (or neglect) all occurrences of un-picked atoms in actions, initial state and goal
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- Which pattern $X$ to pick?
  - $X = g \Rightarrow h_{abs}(s)$ measures distance to goal, like $h_{euc}(s)$
  - Pick $X$ in a way that interconnected components in state space collapse
    - Example: Agent can freely move on a graph of waypoints
      $\Rightarrow$ Abstract away position of the agent
Abstraction heuristics in practice

Realizations of abstraction heuristics:
- Merge-and-shrink abstraction [HHHN14]
- Structural-pattern abstraction [KD09]

General properties:
- Hard part is to identify a good pattern $X$
- Can be admissible, if relaxed plan optimal
- State space explodes (again) if using too big $X$
Critical Path Heuristics

- Suppose we can apply multiple actions in parallel
  - Build *causal dependency graph* as DAG from initial state to goal
  - (At least) one path in DAG is the longest $\Rightarrow$ Critical path
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![Diagram of causal dependency graph]

- Idea for relaxation: Allow applying \( \leq m \) actions in parallel
The $h^m$ Heuristics Family

- Recursive definition of functions $\{ h^m \mid m = 1, 2, \ldots \}$ [GH00], simplified to uniform action costs:

$$h^m(s, g) = \begin{cases} 
0, & \text{if } g \subseteq s. \\
\min_{a \in \text{Rel}(g)} \left( 1 + h^m(s, \gamma^{-1}(g, a)) \right), & \text{if } |g| \leq m. \\
\max_{g' \subseteq g, |g'| \leq m} h^m(s, g'), & \text{otherwise.}
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- $\text{Rel}(g)$: Relevant actions which have atoms in $g$ as effects
- 3rd case picks most costly $m$-subset of goal atoms
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- 3rd case picks most costly $m$-subset of goal atoms
- Pick $m$, then $h_{\text{crit}}(s) := h^m(s, g)$

Properties:
- Admissible for all $m$ (crit. path never longer than optimal seq. plan!)
- Complexity polynomial in problem size for fixed $m$
- Complexity exponential for general $m$ (3rd case explodes)
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Heuristics: Recap

Coarse overview on common heuristics for planning

- Central paradigm: Relaxation
- Heuristics based on Delete-relaxation:
  - Easy heuristics measuring goal distance
  - Fast-Forward heuristic $h^{FF}$
  - Family of Landmark heuristics
- Relaxation by abstraction: Heuristics on pattern databases
- Relaxation by parallelism: Critical paths, family of $h^m$ heuristics
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More resources and information:

- Automated Planning and Acting [GNT16]
- A Beginner’s Introduction to Heuristic Search Planning: Online slides [HR15]
- Don’t be afraid to read some referenced papers!
Heuristics: I can see a pattern there . . .

- **Admissible and accurate** heuristics can be very expensive
  - Example: Abstraction heuristics / Pattern databases

- **Admissible and inexpensive** heuristics are often inaccurate
  - Example: $h^+_d(s)$

- **Accurate and inexpensive** heuristics are hardly admissible
  - Example: “original” $h^{FF}(s)$ with approximated action sets
Heuristics: I can see a pattern there . . .

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  - Example: “original” $h^{FF}(s)$ with approximated action sets

- Is this truly accurate?
  - Possible: Accurate, polynomial complexity, “almost admissible”
  - Contained problems like Hitting Set can be easy in practice (small instances)
Issues with empirical heuristics

Using the benchmarks for inspiration during development, we have been able to come up with a heuristic method that is not probably efficient, but does work well empirically on a large class of planning tasks. — J. Hoffmann et al., 2001 [HN01]

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  ⇒ Feedback loop: Can lead to overfitting effects
    - Heuristics will perform well on considered problems
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  ⇒ **Feedback loop**: Can lead to overfitting effects
    - Heuristics will perform well on considered problems
    - Even small adjustments in some problems may have “unjustifiable” performance impact
- How to avoid overfitting?
  - Try many different planning domains and problems
  - Have some (theoretical) justification for why the heuristic performs good (or bad) on problems
The portfolio approach

- Many heuristics are good on some planning domains, bad on others
- Take this as an opportunity: Why not use all of them?
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- **Automated Planning with a portfolio**
  - Initialize multiple planners searching the same problem, with different heuristics
  - Exploit multi-core architectures to let all planners run in parallel
  - “First one wins” (or: later solutions may improve plan)
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- Pros:
  - Allows “blind” use of specialized heuristics for general problems
  - Easy way to exploit parallelism
The portfolio approach

- Many heuristics are good on some planning domains, bad on others
- Take this as an opportunity: Why not use all of them?
- **Automated Planning with a portfolio**
  - Initialize multiple planners searching the same problem, with different heuristics
  - Exploit multi-core architectures to let all planners run in parallel
  - “First one wins” (or: later solutions may improve plan)

- **Pros:**
  - Allows “blind” use of specialized heuristics for general problems
  - Easy way to exploit parallelism

- **Cons:**
  - Space use linear in amount of workers (if done naïvely)
  - May scale badly with a lot of cores
Stay tuned!

Next lecture: STRIPS Algorithm, Lifted Planning, Plan Space Search


