Up to **four** *people can work on an exercise together. But each of you should be able to explain the solutions to the TA (Bremser). Write your names* **and** *the name of your group (time, TA) on the sheets. Staple them together*

Assignment 8

Deadline: January 19, 2003

Solve any 2 of the 3 exercises.

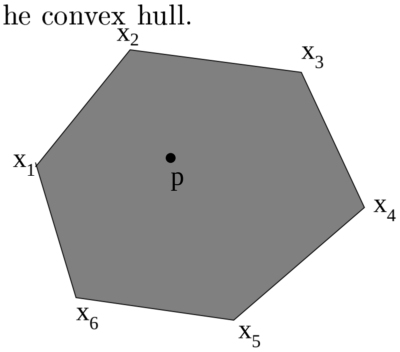
Exercise 1

(Intersection of 2 line segments) Let $\overline{ab}$ and $\overline{cd}$ be two line segments given by their endpoints i.e., we are given the points a, b, c and d.

- Show that it is possible to determine whether these line segments intersect, simply by applying an appropriate combination of orientation tests.
- Suppose these segments do intersect. Then determine the point of intersection. (Use the parametric representation used in the lecture that any point on the line segment $\overline{ab}$ can be written as $sa + (1-s)b$ for $0 \leq s \leq 1$. As s goes from 0 to 1, we traverse the segment from the endpoint a to the endpoint b.)

Exercise 2

The above concept that any point on the segment $\overline{ab}$ can be written as $sa + (1-s)b$ for $0 \leq s \leq 1$ can be generalized. A **convex combination** of points $x_1, x_2, ..., x_n$ is a point of the form $s_1x_1 + s_2x_2 + \cdots + s_nx_n$ where each $s_i \geq 0$ and $s_1 + s_2 + \cdots s_n = 1$. So, this means that every point on the segment $\overline{ab}$ is a convex combination of a and b. Using the fact that every point on a segment is a convex combination of its endpoints,
show that every point in the convex hull of a set of n points can be written as a convex combination of the vertices of the convex hull.



Hint: First, show this property for the points on the boundary. Then show that any interior point, like the point p in the figure, lies on a segment such that one endpoint of the segment is a vertex of the convex hull and the other endpoint is a point on the boundary. This means that p is a convex combination of those two points. Check that this implies that p is a convex combination of the vertices of the convex hull.

Exercise 3

Let S_1 be a set of n disjoint horizontal line segments and let S_2 denote a set of m disjoint vertical line segments. Give an $O((n+m)\log(n+m))$ time algorithm to count how many intersection points there are in $S_1 \cup S_2$.