Up to **four** *people can work on an exercise together. But each of you should be able to explain the solutions to the TA (Bremser). Write your names* **and** *the name of your group (time, TA) on the sheets. Staple them together*

Assignment 3

Deadline: November 17, 2003

Solve at least two of the following three exercises completely (or more of them partially)

Exercise 1

Arbitrage is the use of discrepancies in currency exchange rates to transform one unit of a currency into more than one unit of the same currency. For example, suppose that 1 U.S. dollar buys 0.7 British pound, 1 British pound buys 9.5 French francs, and 1 French franc buys 0.16 U.S. dollar. Then by converting currencies, a trader can start with 1 U.S. dollar and buy $0.7 \times 9.5 \times 0.16 = 1.064$ U.S. dollars, thus turning a profit of 6.4 percent.

Suppose that we are given n currencies $c_1, c_2, \ldots, c_n$ and $n \times n$ table R of exchange rates, such that one unit of currency c_i buys $R[i,j]$ units of currency c_j.

(a) Give an efficient algorithm to determine whether or not there exists a sequence of currencies $\langle c_{i_1}, c_{i_2}, \ldots, c_{i_k} \rangle$ such that

$R[i_1,i_2] \cdot R[i_2,i_3] \cdots R[i_{k-1},i_k] \cdot R[i_k,i_1] > 1$.

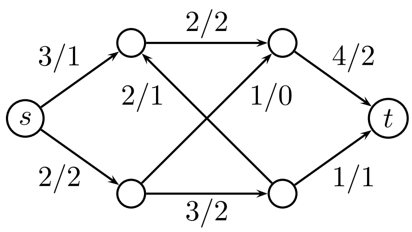
Analyze the running time of your algorithm.

(b) Give an efficient algorithm to print out such a sequence if one exists. Analyze the running time of your algorithm.

Hint: $\prod_{i=1}^{k} x_i = e^{\sum_{i=1}^{k} \ln x_i}$. Furthermore, $\prod_{i=1}^{k} x_i > 1$ iff $1/\prod_{i=1}^{k} x_i < 1$.

Exercise 2

Consider the graph G and the flow f given on the right.



(a) Is f a blocking flow?

(b) Give the residual graph G_f.

(c) Give the layered subgraph L_f of G_f.

(d) Find an augmenting path and give the resulting augmented flow. Repeat until the flow is maximum.

(e) Give a saturated (s,t)-cut for the maximum flow.

(*continues on the next page*)

Exercise 3
Which of the following claims are true and which are false. Justify your answer by giving either a (short) proof or a counterexample.

(a) In any maximum flow there are no cycles that carry positive flow. (A cycle $\langle e_1, \ldots, e_k \rangle$ carries positive flow iff $f(e_1) > 0, \ldots, f(e_k) > 0$.)

(b) There always exists a maximum flow without cycles carrying positive flow.

(c) If all edges in a graph have distinct capacities, there is a unique maximum flow.

(d) In a directed graph with at most one edge between each pair of vertices, if we replace each directed edge by an undirected edge, the maximum flow value remains unchanged.

(e) If we multiply all edge capacities by a positive number λ, the minimum cut remains unchanged.

(f) If we add a positive number λ to all edge capacities, the minimum cut remains unchanged.

(g) If we add a positive number λ to all edge capacities, the maximum flow increases by a multiple of λ.