Up to **four** *people can work on an exercise together. But each of you should be able to explain the solutions to the TA (Bremser). Write your names* **and** *the name of your group (time, TA) on the sheets. Staple them together*

Assignment 4

Deadline: November 24, 2003

Solve at least two of the following four exercises completely (or more of them partially)

Exercise 1
A spammer is located at one node q in a undirected communication network G and peaceful email users are located at nodes denoted by the set S. Let c_{uv} denote the effort required to install a spam filter for the network edge (u, v). The problem is to determine the minimal effort required to isolate the spammer from the email users using the spam filters. Find an efficient polynomial time algorithm to solve this problem.

Exercise 2
We define a *most vital arc* of a network as an arc whose deletion causes the largest decrease in the maximum s-t-flow value. Let f be an arbitrary maximum s-t-flow. Either prove the following claims or show through counterexamples that they are false:

(a) A most vital arc is an arc e with the maximum value of $c(e)$.

(b) A most vital arc is an arc e with the maximum value of $f(e)$.

(c) A most vital arc is an arc e with the maximum value of $f(e)$ among arcs belonging to some minimum cut.

(d) An arc that does not belong to some minimum cut cannot be a most vital arc.

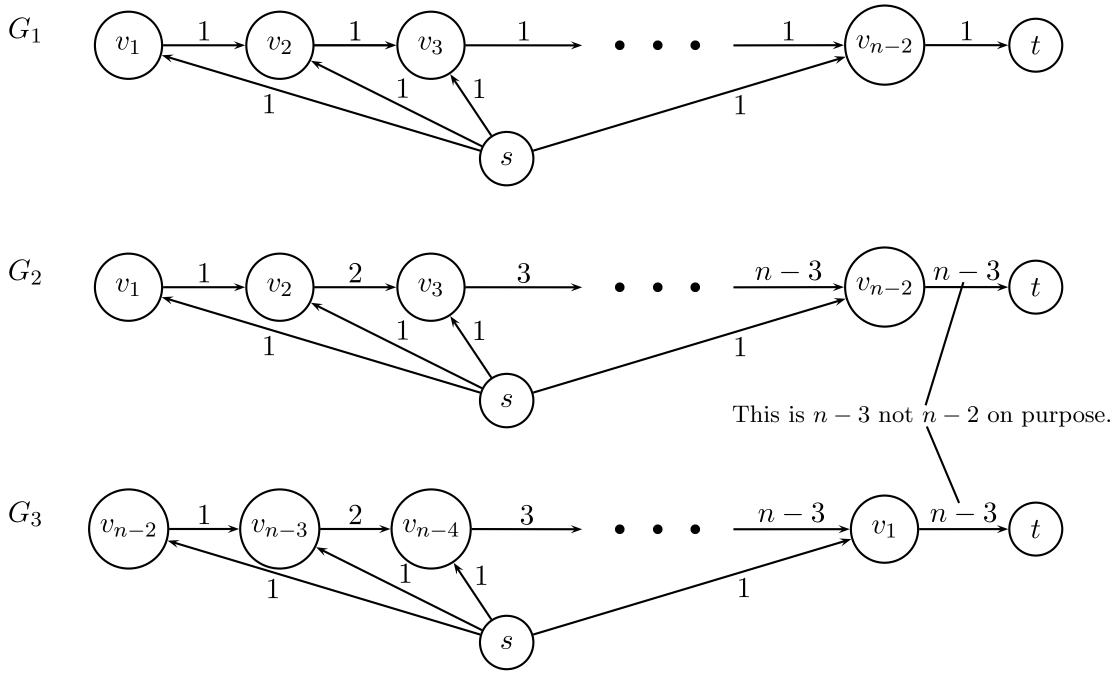
(e) A network might contain several most vital arcs.

Exercise 3
Dining Problem. Several families go out to dinner together. To increase their social interaction, they would like to sit at tables so that no two members of the same family are at the same table. Show how to formulate finding a seating arrangement that meets this objective as a maximum flow problem. Assume that the dinner contingent has p families and that the ith family has $a(i)$ members. Also assume that q tables are available and that the jth table has a seating capacity of $b(j)$.

Exercise 4
In this exercise we analyze the running time of different variations of the preflow-push algorithm with the highest-level selection rule. Assume that ties are broken as follows:

- Of active nodes on the highest level the one with the smallest number is selected first.
- Of eligible edges the one leading to the largest numbered node is selected first.
  (Would a different ordering of the eligible edges change your answer in any of the cases?)

Consider the following capacitated graphs:



Give the asymptotic running time (total number of pushes and relabelings) of the following three heuristics for all three graphs. Justify your answer. (Hint: The running time is $\Theta(n)$ or $\Theta(n^2)$ in all cases.)

(a) *Local relabeling.* When relabeling a node v, the new level is computed as $d(v) = 1 + \min\{d(w) \mid (v,w) \in G_f\}$.

(b) *Global relabeling.* In the beginning and after every m ($= 2n-4$ in this case) operations, the distance labels are recomputed as follows:

$$d(v) = \begin{cases} \mu(v,t) & \text{if there is a path from } v \text{ to } t \text{ in } G_f \\ n + \mu(v,s) & \text{if there is a path from } v \text{ to } s \text{ but not to } t \text{ in } G_f \\ 2n-1 & \text{otherwise} \end{cases}$$

Here $\mu(v,w)$ is the smallest number of edges on a path from v to w.

(c) *Gap heuristic.* When a level ℓ becomes empty, all nodes on the levels $\ell+1$ to $n-1$ are lifted to level n.