Karlsruhe Institute of Technology

Text Indexing

Lecture 01: Tries

Florian Kurpicz

String Dictionary

Given a set $S \subseteq \Sigma^\star$ of prefix-free strings, we want to answer:

- is $x \in \Sigma^\star$ in S
- add $x \notin S$ to S
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Given a set $S = \{S_1, \ldots, S_k\}$ of prefix-free strings, a trie is a labeled rooted tree $G = (V, E)$ with:

1. k leaves
2. $\forall S_i \in S$ there is a path from the root to a leaf, such that the concatenation of the labels is S_i
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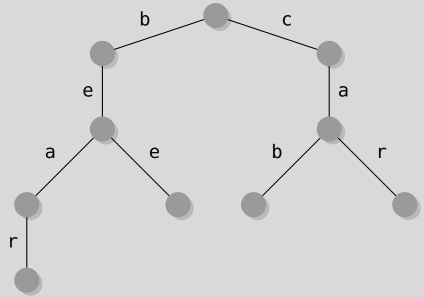

$S = \{\mathtt{bear}, \mathtt{bee}, \mathtt{cab}, \mathtt{car}\}$

Queries: Insert, Contains, and Delete a Pattern

Same for all

- start at root and follow existing children

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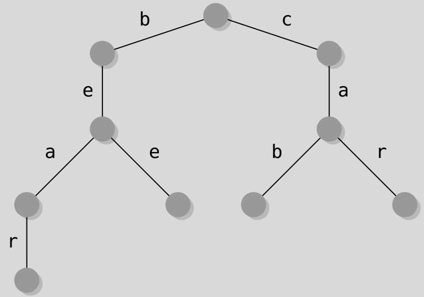
- insert rest of pattern ℹ prefix-free



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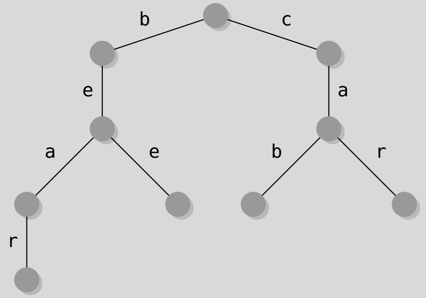

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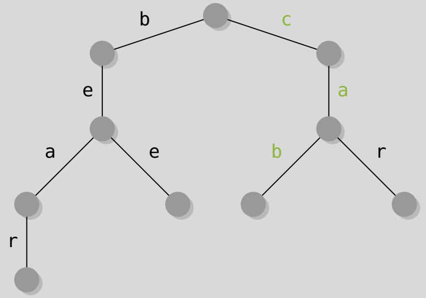

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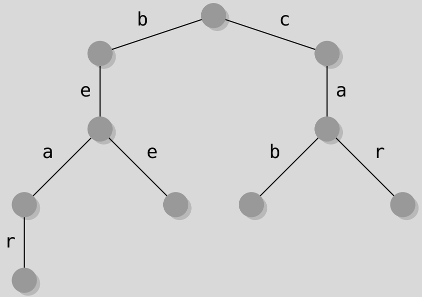

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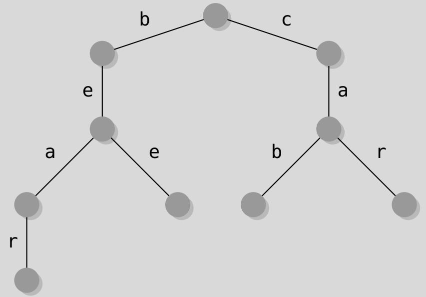

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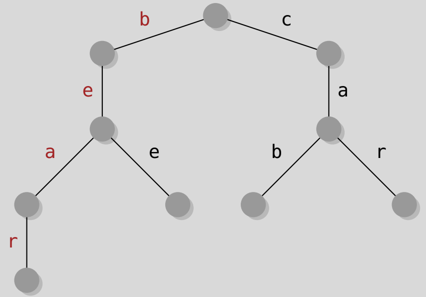

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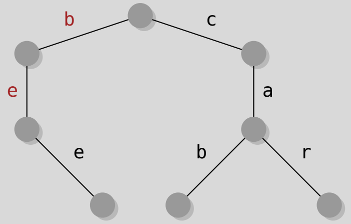

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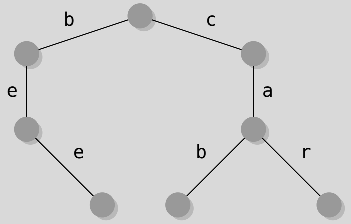

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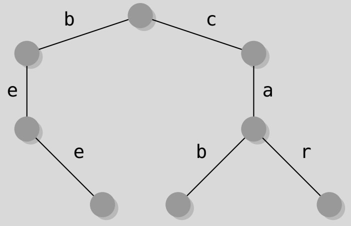

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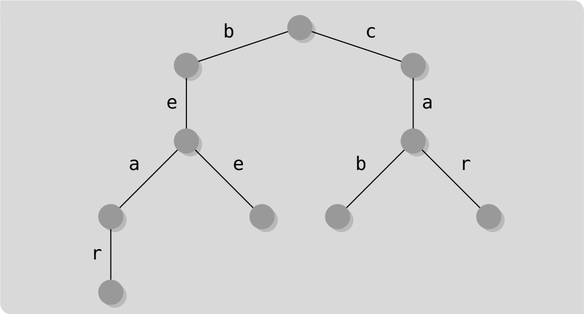
- is `cab` in S
- remove `bear` from S
- how can we find the predecessor of `can`?

Why Prefix-Free

- insert `beer`



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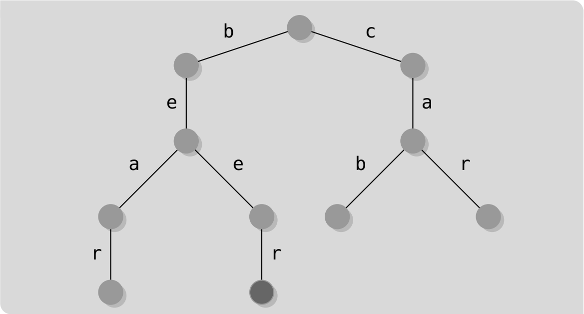

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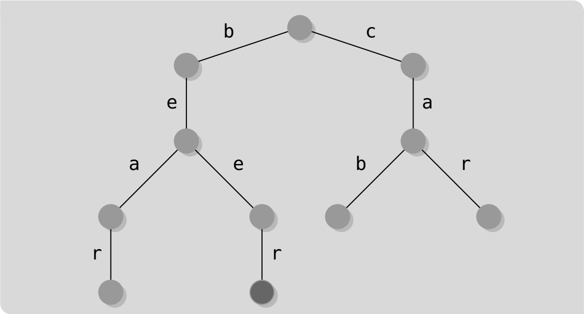

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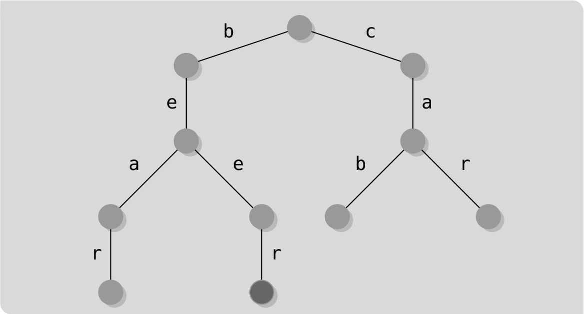

Why Prefix-Free

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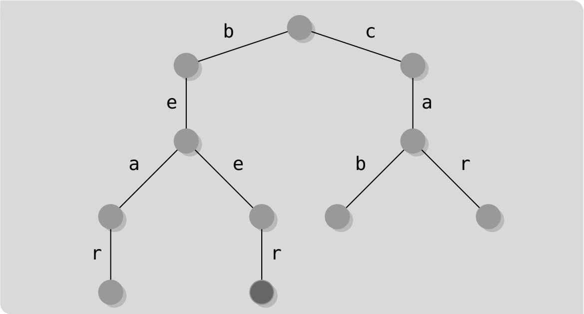
- remember which node refers to a string
- or (much preferred) make strings prefix free



Next Steps

Setting

- alphabet Σ of size σ
- k strings $\{s_1, \dots, s_k\}$ over the alphabet Σ
- total size of strings is $N = \sum_{i=1}^{k} |s_i|$
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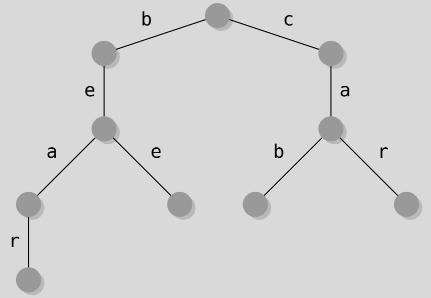

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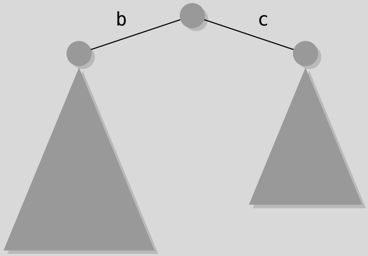

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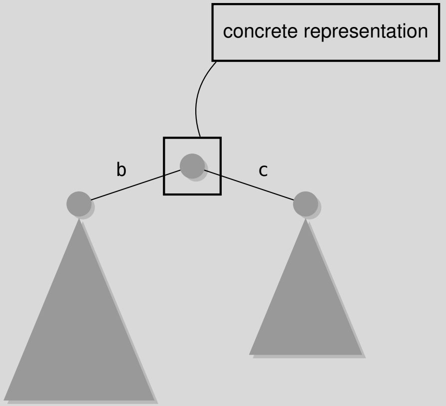
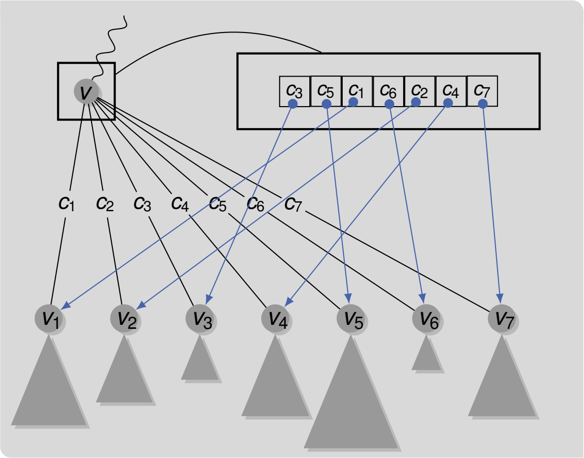


Arrays of Variable Size

- store children (character and pointer) in the order they are added
- to find child scan array
- to delete child swap with last and remove last
  - ℹ children are not ordered



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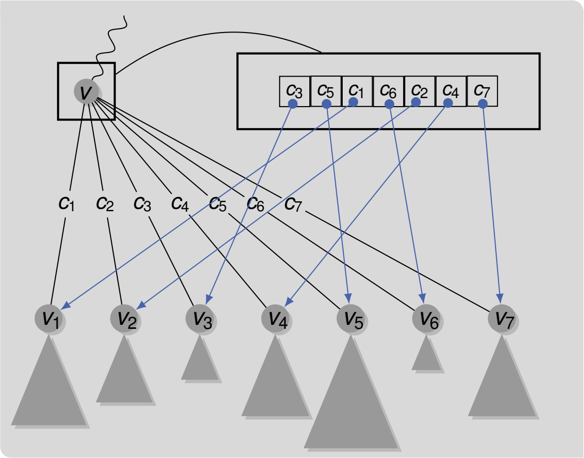

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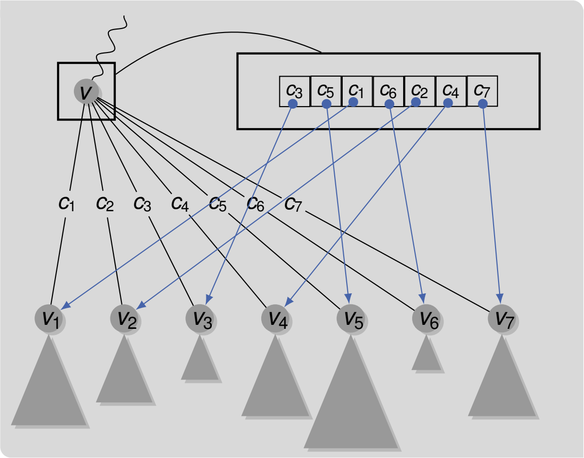

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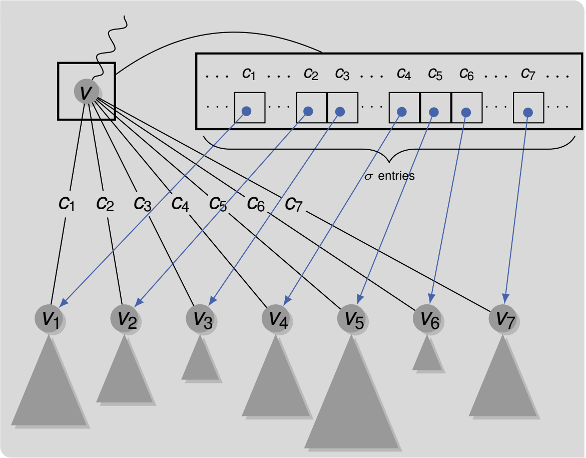

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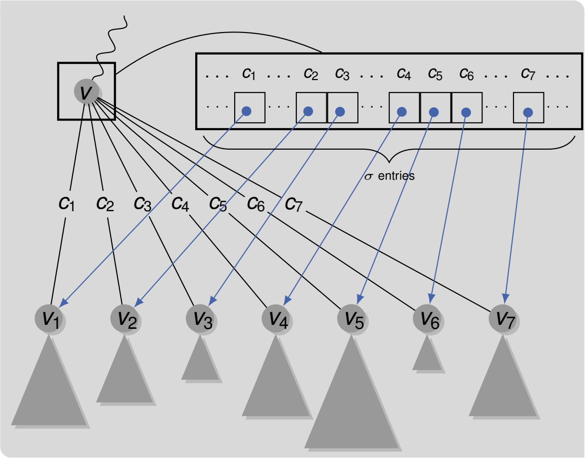

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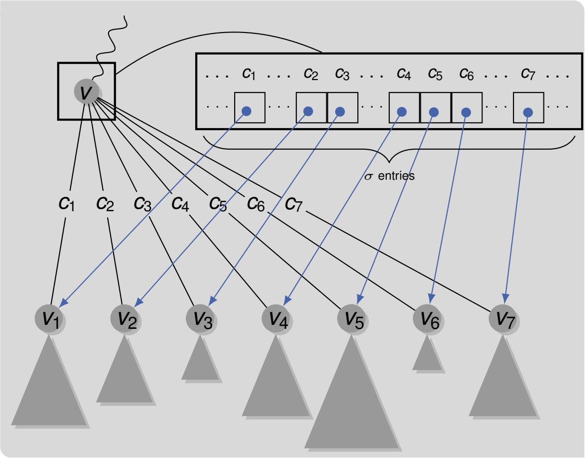
Space

- $O(N \cdot \sigma)$ words ℹ very bad



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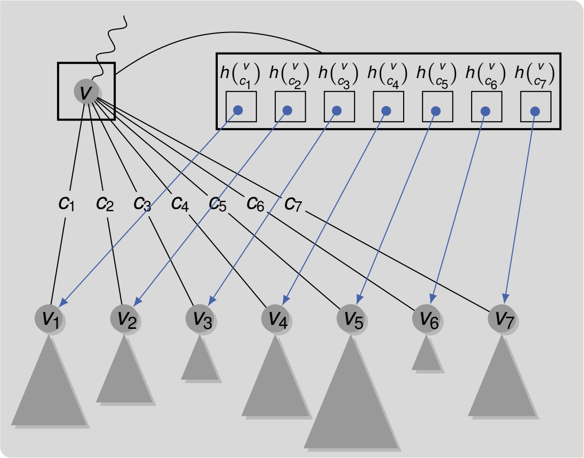
- or use global hash table for whole trie



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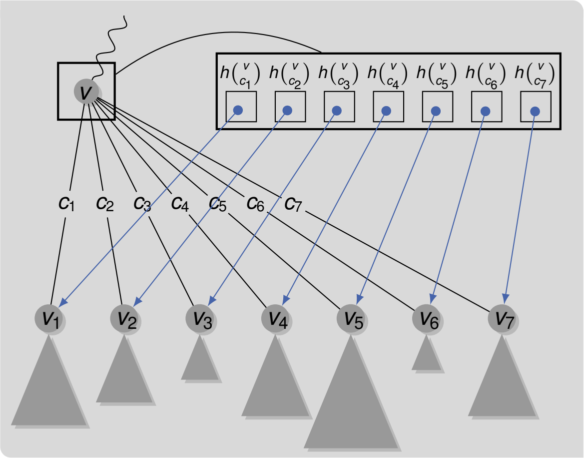

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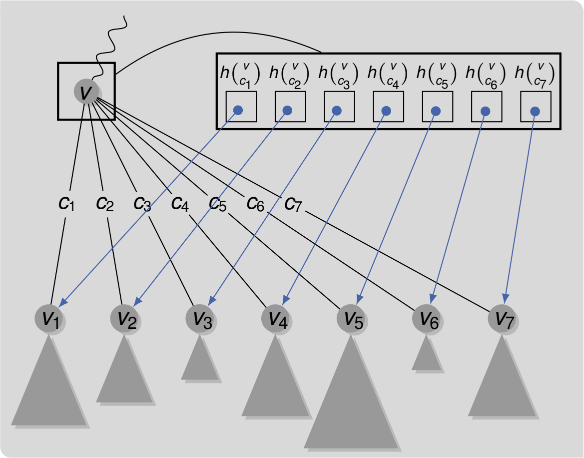

Balanced Search Trees

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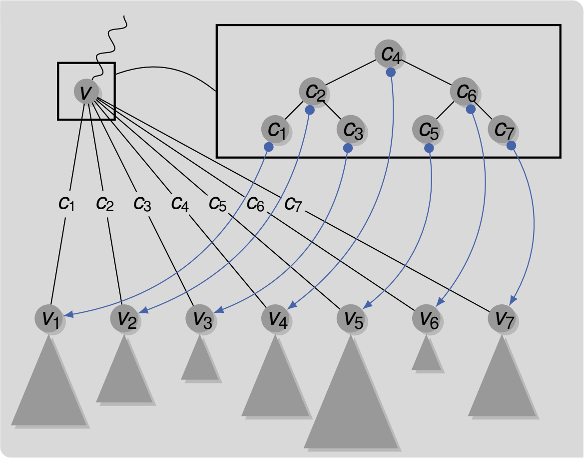
- e.g., AVL tree, red-black tree, . . .
- in static setting sorted array and binary search



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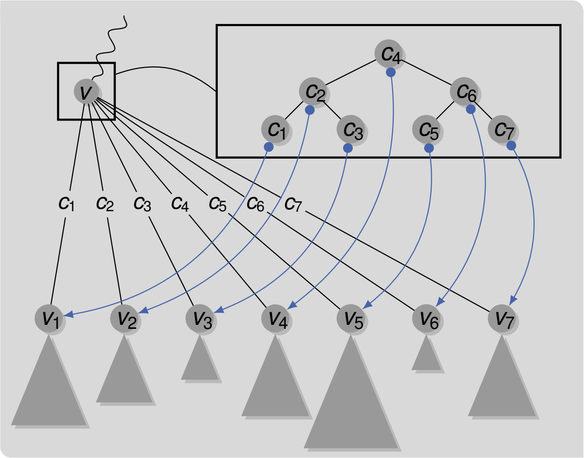

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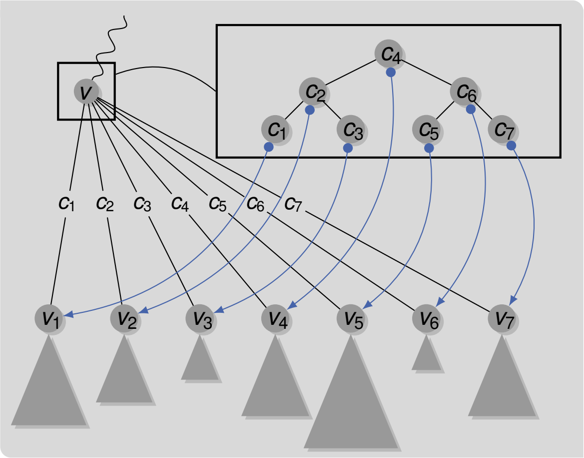
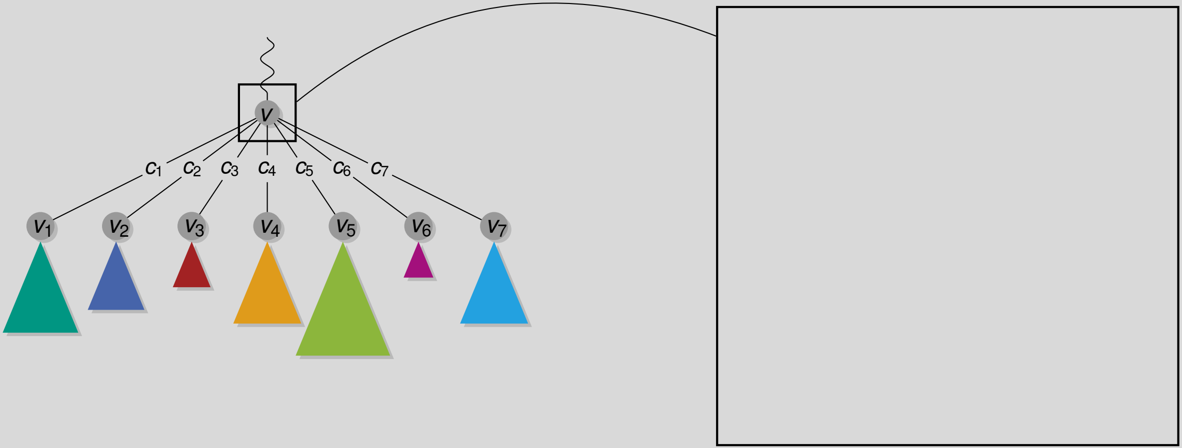


Weight-Balanced Search Trees (1/2)



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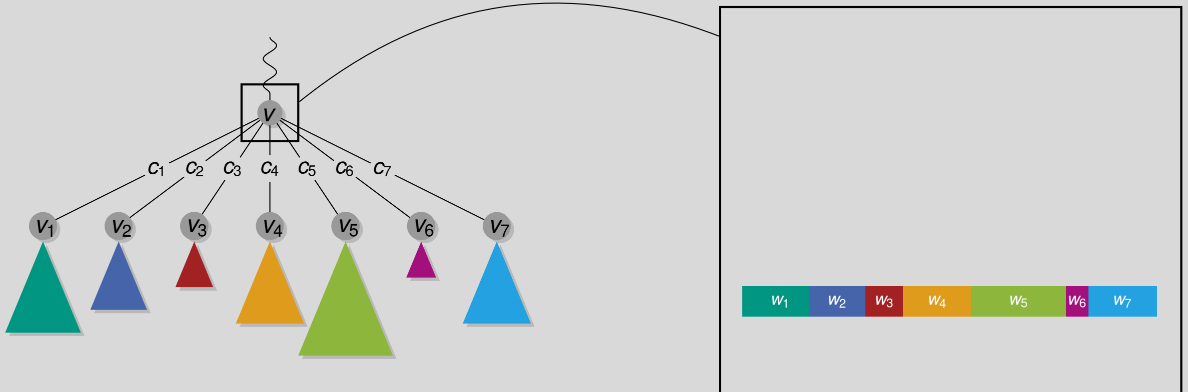

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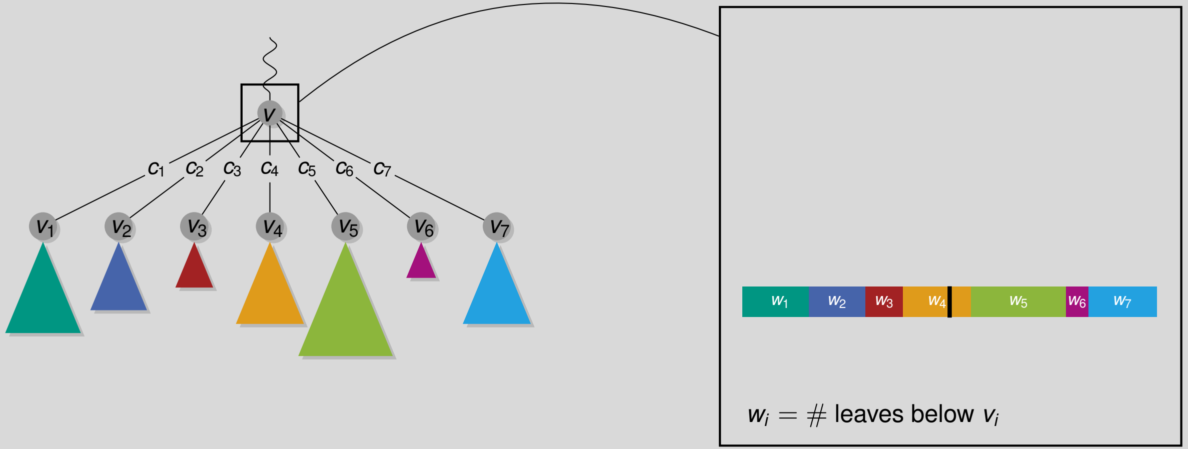

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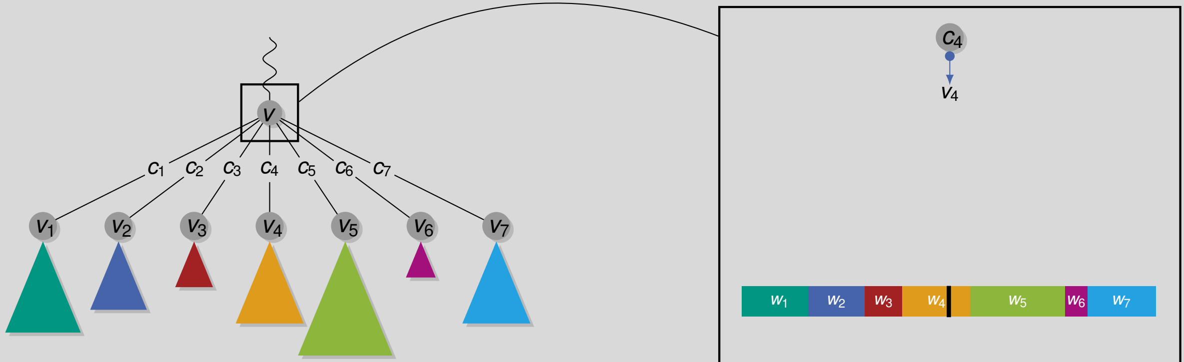

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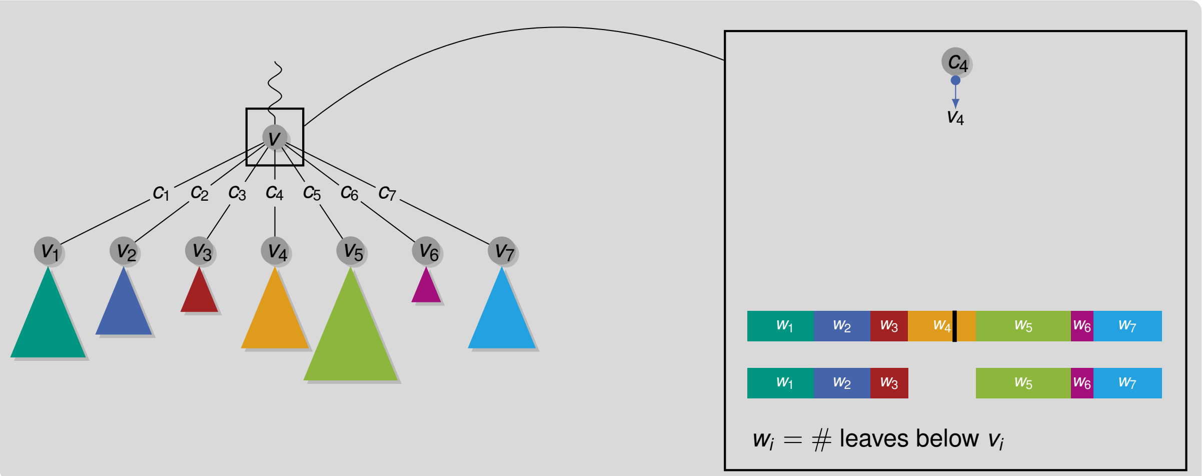

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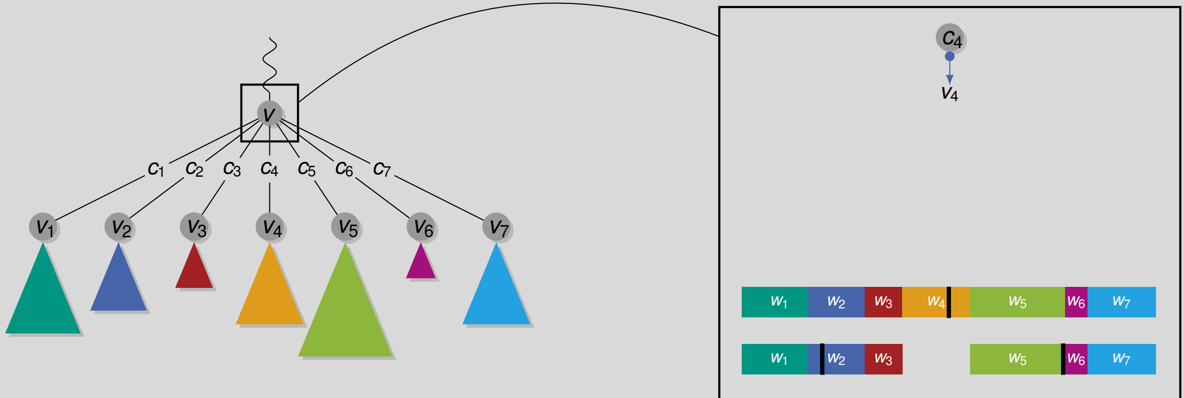

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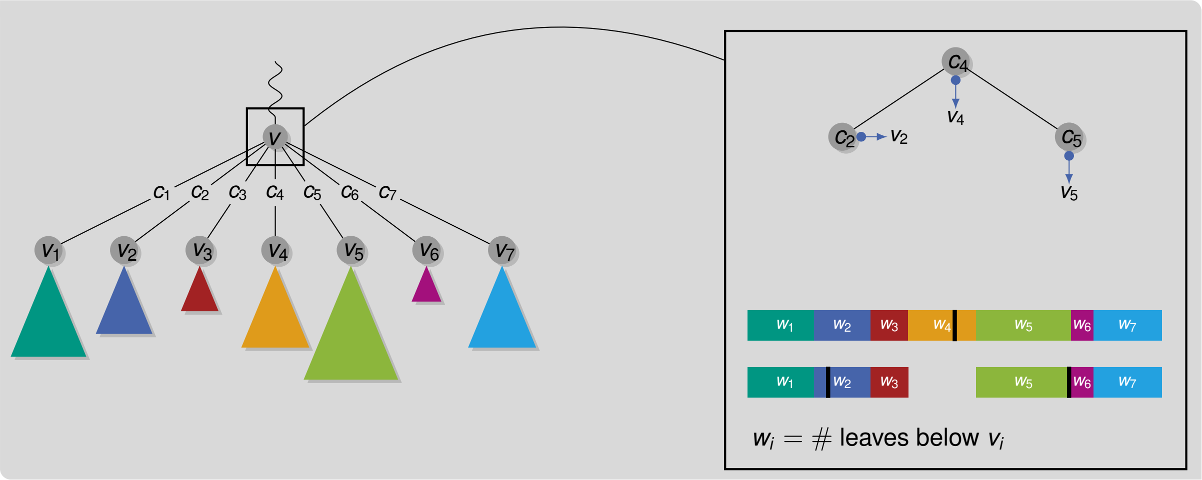

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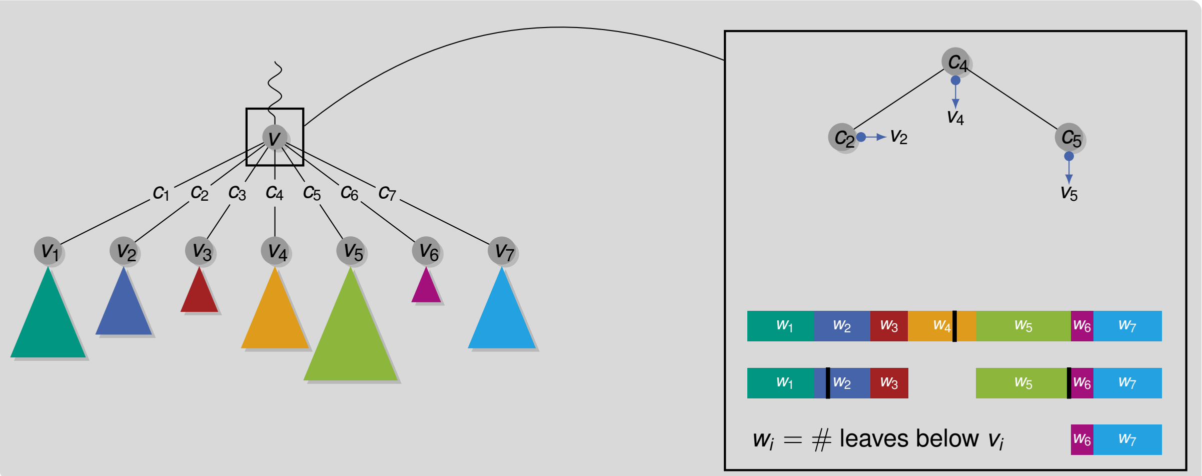

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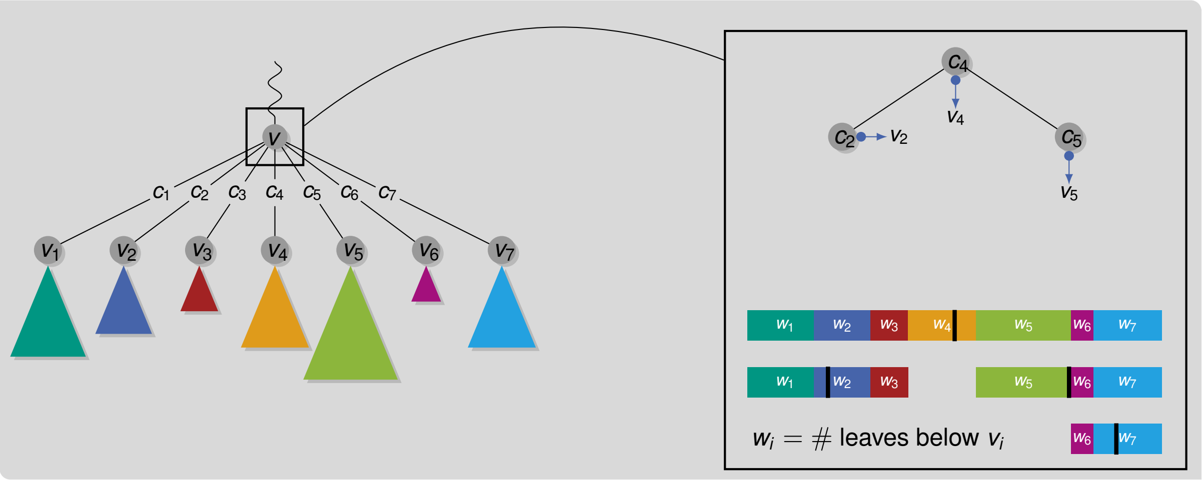

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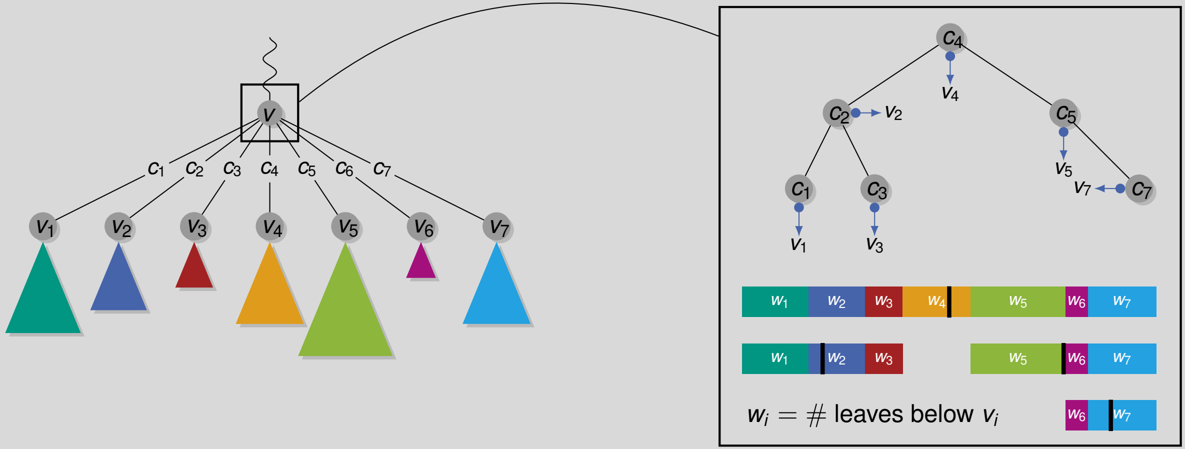

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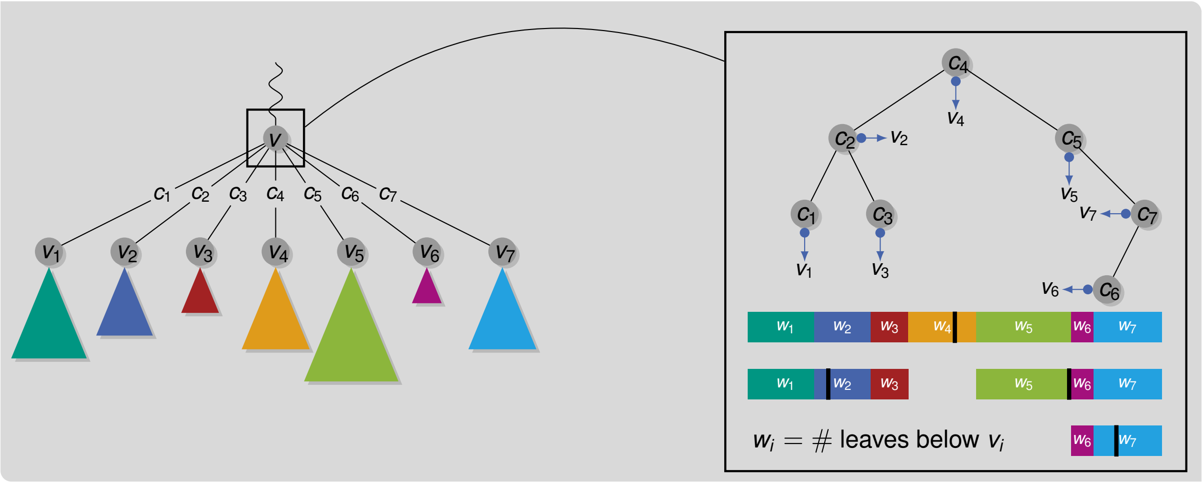

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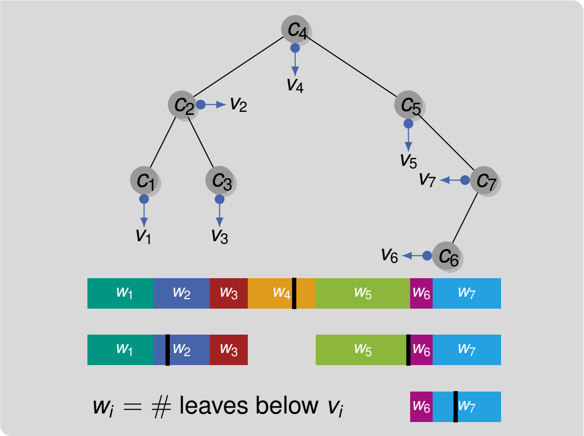

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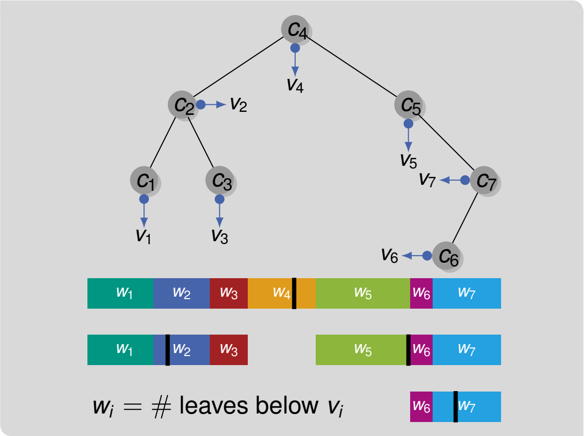
- or halve number of strings



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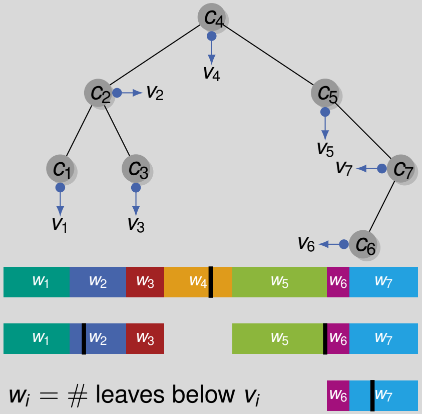
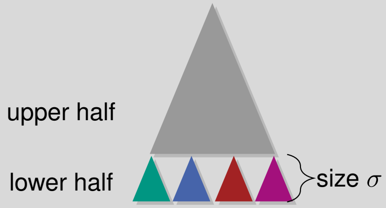


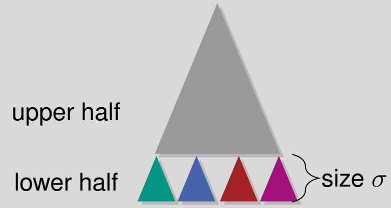
Two-Levels with Weight-Balanced Search Trees

- split tree into upper and lower half
- lower half deepest nodes such that subtrees have size $O(\sigma)$
- weight-balanced search trees for lower half
- fixed-size arrays in upper half ℹ branching nodes only



Two-Levels with Weight-Balanced Search Trees

- split tree into upper and lower half
- lower half deepest nodes such that subtrees have size $O(\sigma)$
- weight-balanced search trees for lower half
- fixed-size arrays in upper half ℹ branching nodes only



Query Time (Contains)

- upper half: $O(m)$
- lower half: $O(m + \lg \sigma)$
- total: $O(m + \lg \sigma)$

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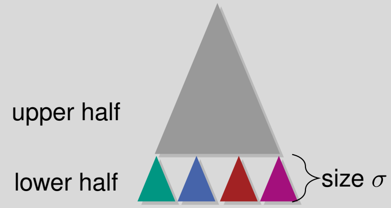

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Space

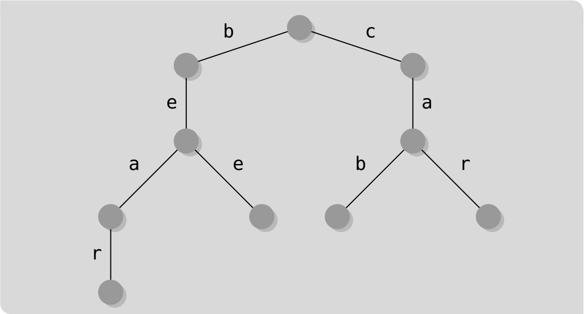
- upper half: $O(N)$ words
  ℹ $O(N/\sigma)$ branching nodes
- lower half: $O(N)$ words
- total: $O(N)$ words

Theoretical Comparison

Representation	Query Time (Contains)	Space in Words
arrays of variable size	$O(m \cdot \sigma)$	$O(N)$
arrays of fixed size	$O(m)$	$O(N \cdot \sigma)$
hash tables	$O(m)$ w.h.p.	$O(N)$
balanced search trees	$O(m \cdot \lg \sigma)$	$O(N)$
weight-balanced search trees	$O(m + \lg k)$	$O(N)$
two-levels with weight-balanced search trees	$O(m + \lg \sigma)$	$O(N)$

Compact Trie

- tries have unnecessary nodes
- branchless paths can be removed
- edge labels can consist of multiple characters



Compact Trie

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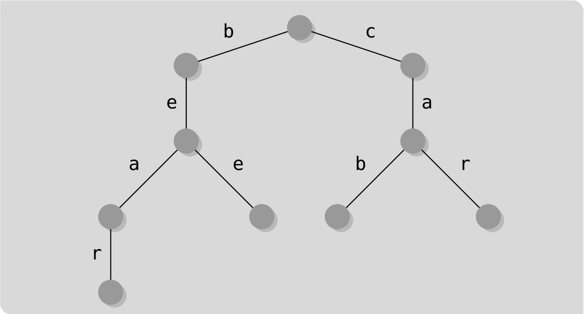
Definition: Compact Trie

- A compact trie is a trie where all branchless paths are replaced by a single edge.
- The label of the new edge is the concatenation of the replaced edges' labels.



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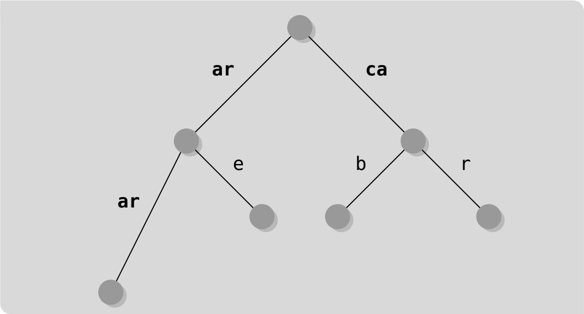

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