Karlsruhe Institute of Technology

Text Indexing

Lecture 11: Top-k Document Retrieval

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PINGO

https://pingo.scc.kit.edu/776916

Recap: Inverted Index and List Encodings

Definition: Inverted Index

Given a set of documents and terms that are contained in the documents, an inverted index stores the terms and associated with each term t

- the number of documents f_t that contain t and
- an ordered list $L(t)$ of documents containing t

```
1 The old night keeper keeps the keep in the town
2 In the big old house in the big old gown
3 The house in the town had the big old keep
4 Where the old night keeper never did sleep
5 The night keeper keeps the keep in the night
6 And keeps in the dark and sleeps in the light
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term t	f_t	$L(t)$
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dark	1	[6]
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had	1	[3]
house	2	[2, 3]
in	5	[1, 2, 3, 5, 6]
...	...	...

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List Encodings

- Δ-encoding
- unary- and ternary-encoding
- Elia-γ and -δ-encoding
- Golomb- and Fibonacci-encoding

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Document Listing

- similar to last lecture
- get all documents containing a *phrase*

Definition: Document Listing

Given a collection of D documents $\mathcal{D} = \{d_1, d_2, \ldots, d_D\}$ containing symbols from an alphabet $\Sigma = [1, \sigma]$ and a pattern $P \in \Sigma^*$, return all $j \in [1, D]$, such that d_j contains P.

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- $d_1 =$ ATA
- $d_2 =$ TAAA
- $d_3 =$ TATA

And for queries:

- $P =$ TA is contained in d_1, d_2, and d_3
- $P =$ ATA is contained in d_1 and d_3

Basic Concepts

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Definition: Document Concatenation

Given a collection of D documents $\mathcal{D} = \{d_1, d_2, \ldots, d_D\}$ containing symbols from an alphabet $\Sigma = [1, \sigma]$ where each document ends with a special symbol $\# \notin \Sigma$, the string

$$\mathcal{C} = d_1 d_2 \ldots d_D \$$$

is called the concatenation of the documents with $\$ \notin \Sigma$ and $\$ < \# < \alpha$ for all $\alpha \in \Sigma$

- $N = |\mathcal{C}| = \sum_{i=1}^{D} |d_i|$

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Document Concatenation:

- ATA#TAAA#TATA#$

Suffix Array for Document Concatenation

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$$DA[i] = j \iff \sum_{k=1}^{j-1} |d_k| < SA[i] \leq \sum_{k=1}^{j} |d_k|$$

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Naive Document Listing

- given document concatenation $\mathcal{C}$, its suffix array *SA*, and document array *DA*
- enhance suffix array to do pattern matching in $O(|P|)$ time ℹ only briefly discussed in lecture
- find interval in suffix array matching *P*
- report all documents in interval in *DA*
- problem: $O(|P| + N)$ query time ℹ very bad

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- is there a better solution?
- better query time
- better (or at least equal) space requirements?

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Optimal Time Document Listing (1/2) [Mut02]

Definition: Chain Array

Given document concatenation $\mathcal{C}$, its suffix array SA, and document array DA, the **chain array** CA is defined as

$$CA[i] = \max\{j < i \colon DA[j] = DA[i]\} \cup \{0\}$$

- chains same documents together
- find lexicographically smaller suffix of same document
- use it to report documents just once
- build RMQ data structure for CA

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Optimal Time Document Listing (2/2)

- given document concatenation $\mathcal{C}$, its suffix array *SA*, document array *DA*, and chain array *CA* with RMQ data structure
- find interval $SA[s, e]$ as before
- report document $DA[m]$ only if $CA[m] < s$ for $m \in [s, e]$

- find all positions where $CA[m] < s$ with RMQs
- get arg min of *CA* in interval and report $DA[m]$ if $CA[m] < s$
- split interval in $[s, m-1]$ and $[m+1, e]$ and recurse
- ignore intervals where nothing is reported

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
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Lemma: Optimal Document Listing

Listing all documents containing a pattern P can be done in $O(|P| + occ)$ time

Top-k Document Retrieval for Single-Term Frequencies

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Definition: Top-k Document Retrieval

Given a collection of D documents $\mathcal{D} = \{d_1, d_2, \ldots, d_D\}$ containing symbols from an alphabet $\Sigma = [1, \sigma]$, a pattern $P \in \Sigma^*$, and a threshold k , return the top-k documents $j \in [1, D]$, such that d_j contains P most often

- retrieve *occ* distinct documents where P occurs
- determine frequency of P in each document
- maintain min-heap of (frequency,document)-pairs of size k
- total time: $O(|P| + occ(\lg k + \lg N))$

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- can we do better

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- optimal solution: $O(|P| + k)$ query time in $O(N \lg N)$ bits [NN12]
- now: $O(m + k \lg N)$ [GKN17]

Recap: Suffix Tree

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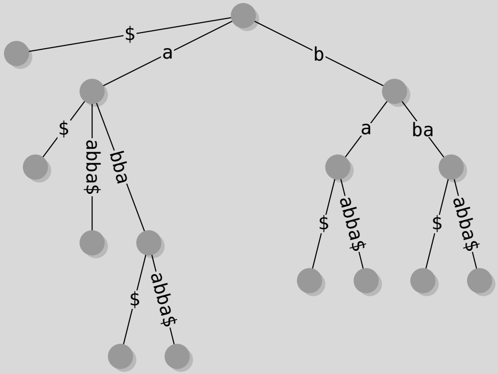
Definition: Suffix Tree [Wei73]

A suffix tree (*ST*) for a text *T* of length *n* is a

- compact trie
- over $S = \{T[1..n], T[2..n], \ldots, T[n..n]\}$
  - suffixes are prefix-free due to sentinel



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a	b	b	a	a	b	b	a	$

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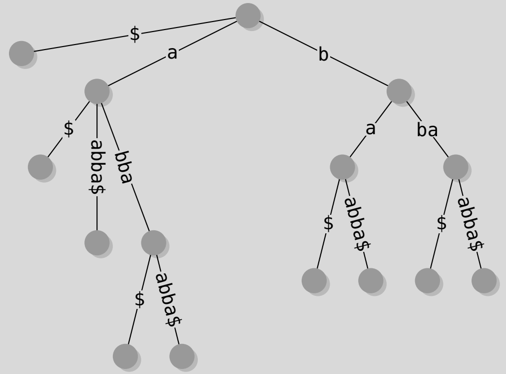
Let $G = (V, E)$ be a compact trie with root r and a node $v \in V$, then

- $\lambda(v)$ is the concatenation of labels from r to v
- $d(v) = |\lambda(v)|$ is the string-depth of v

  ℹ string depth $\neq$ depth



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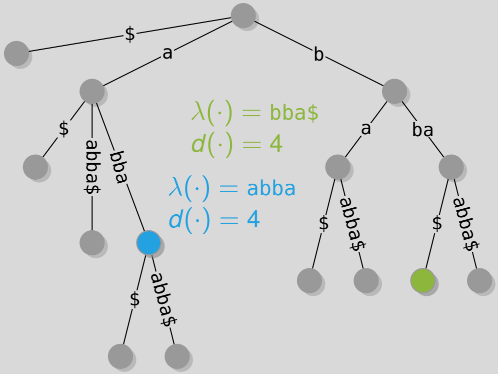

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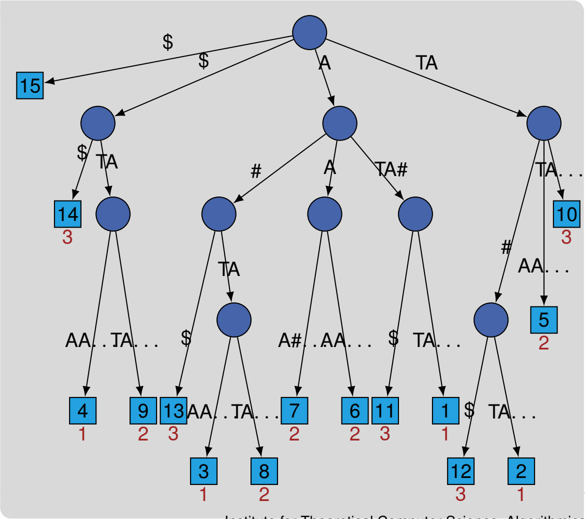
Generalized Suffix Tree for Top-k Document Retrieval (1/4)

- a generalized suffix tree is a suffix tree for a set of strings
- document concatenation is a set of strings

Mark Document Numbers

- mark all leaves with *DA*-entry *i*



Generalized Suffix Tree for Top-*k* Document Retrieval (1/4)

- a generalized suffix tree is a suffix tree for a set of strings
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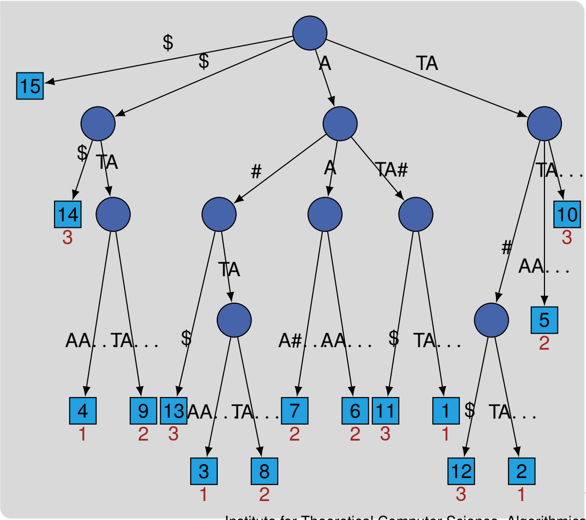
- mark all leaves with *DA*-entry *i*
- add *i* to nodes that are lowest common ancestor of tow leaves marked with *i*



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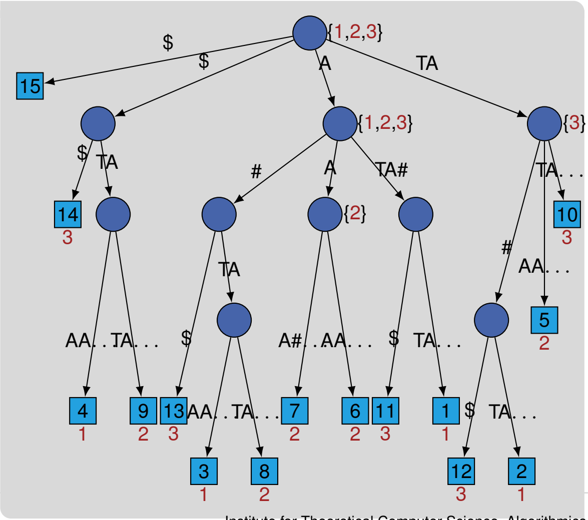

Generalized Suffix Tree for Top-k Document Retrieval (2/4)

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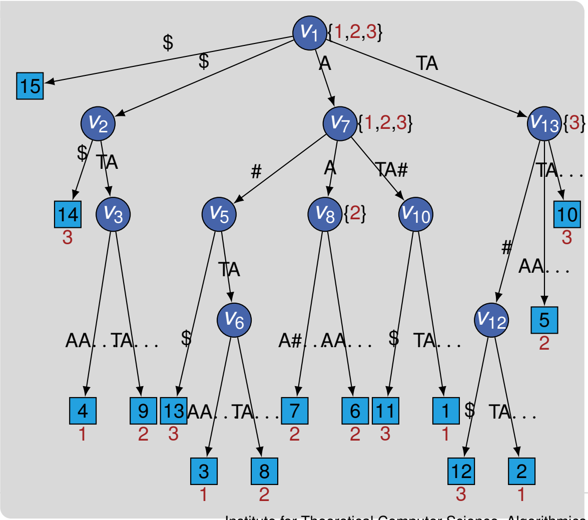
Inner Node Names

- leaf index is rank of suffix in $[1, N]$ in leaf
- each inner node gets v gets $id(v)$, which is the leaf index of rightmost child in leftmost leaf

- $id(v) \neq id(w)$ for all inner nodes $v \neq w$
- $id(v) \in [1, N]$
- $id(v) - 1 \in [lb(v), rb(v),]$, with *interval* $lb(v), rb(v)$ being v's suffix array interval

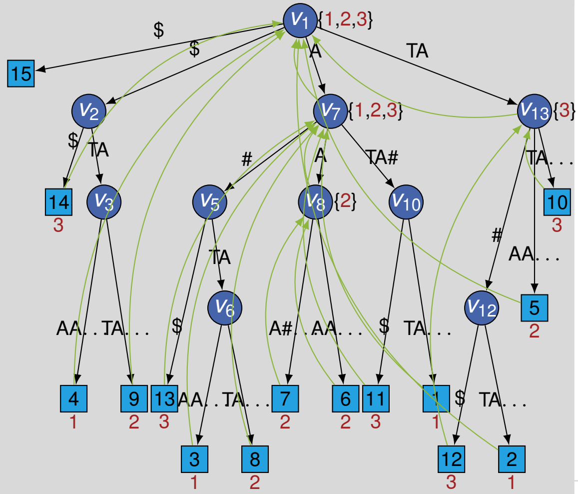
- example on the board



Generalized Suffix Tree for Top-k Document Retrieval (3/4)

- connect node with id i to closest ancestor containing id i

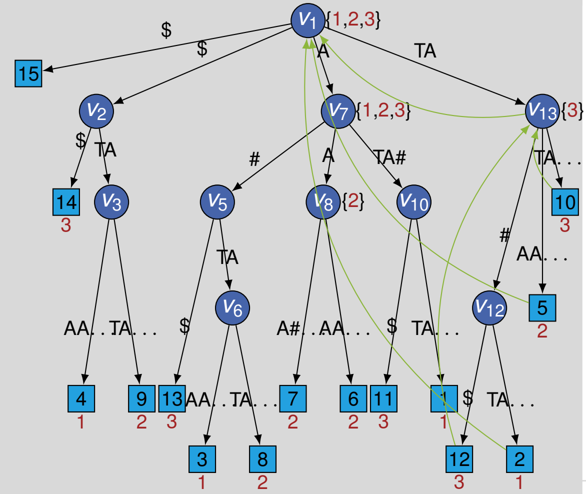
- nodes marked with id i correspond to suffix tree of d_i
- document id i occurs at most $|d_i|$ times in leaves and $|d_i| - 1$ times in inner nodes
- there are at most $O(N)$ document ids in the generalized suffix tree



Generalized Suffix Tree for Top-k Document Retrieval (4/4)

- to retrieve documents containing patter P
- select locus of P ℹ first node v with $\lambda(v)$ is prefix of P

- per document at most one pointer leaves subtree of locus v
- associate each pointer with number of occurrences of documents in pointers source (weight)
- pointer of document i leaving subtree has maximum weight of all document i pointers in subtree
- document listing is listing all documents of pointers leaving subtree



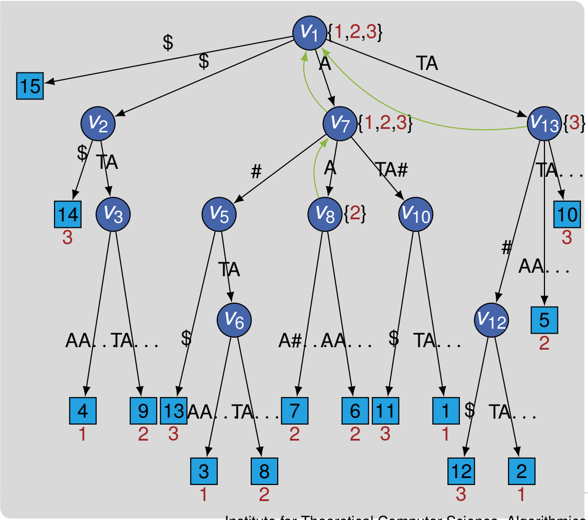
Representing Pointers on a Grid (1/2)

- now: report top-k documents

- represent pointers in a grid
- for simplicity only weights ≥ 2 starting at inner node

- assign each pointer to (x, y)-coordinate
  - x: $id(\text{source})$
  - y: $d(\text{target})$
- each point is associated with pointers weight

- given a locus v, all pointers leaving the subtree have y-coordinate $< d(v)$



Representing Pointers on a Grid (2/2)

- grid can be represented using wavelet tree
- range maximum query for each level

- example range queries in wavelet trees on the board

Answering Queries

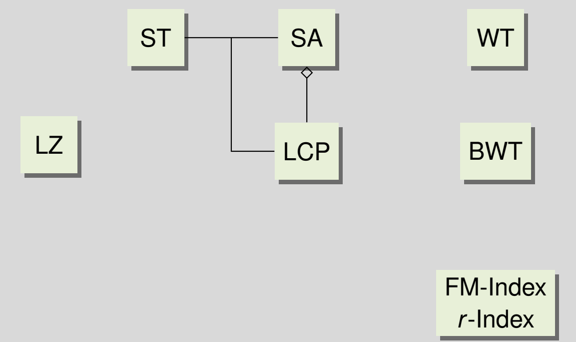
- find string depth of locus in suffix tree
- answer range query in grid
- if represented as wavelet tree, use RMQs on each level to report top-k documents
- if $\leq k$ documents, use document listing
- total time: $O(m + k \lg N)$

Conclusion and Outlook

This Lecture

- document listing
- top-k document retrieval (single term frequency)

Linear Time Construction



Conclusion and Outlook

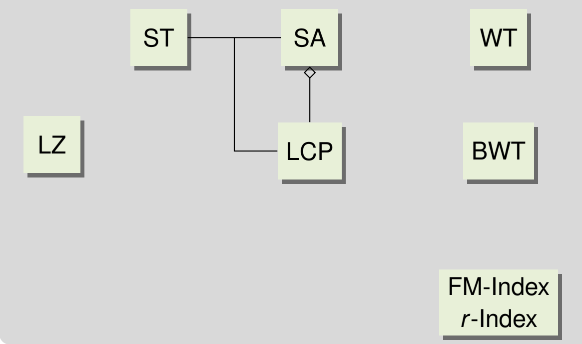
This Lecture

- document listing
- top-k document retrieval (single term frequency)

Next Lecture

- longest common extension queries

Linear Time Construction



Misc

Karlsruhe Institute of Technology

Oral Exam

- registration is open
- is there anybody studying w.r.t. "Prüfungsordnung vor 2015"

Evaluation

https://onlineumfrage.kit.edu/evasys/online.php?p=ZF8QT

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