

# Examples of Massively Parallel Non-Numerical Algorithms

Algorithm Engineering for Parallel Sorting and Graph Generation

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## Sorting Algorithms

One of the most fundamental non-numeric algorithms

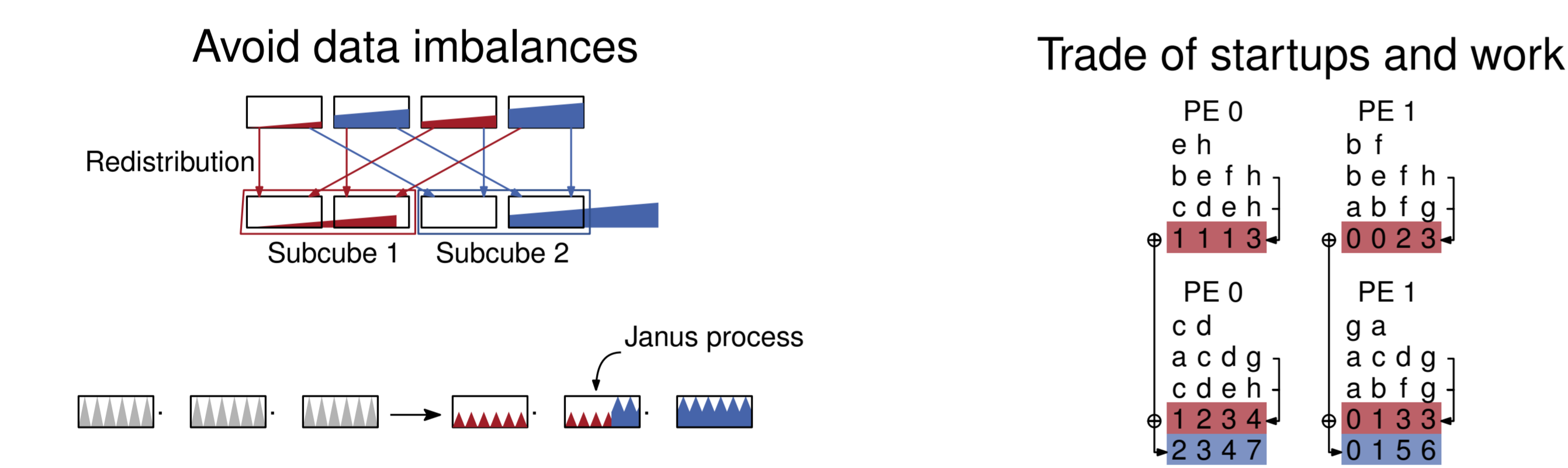
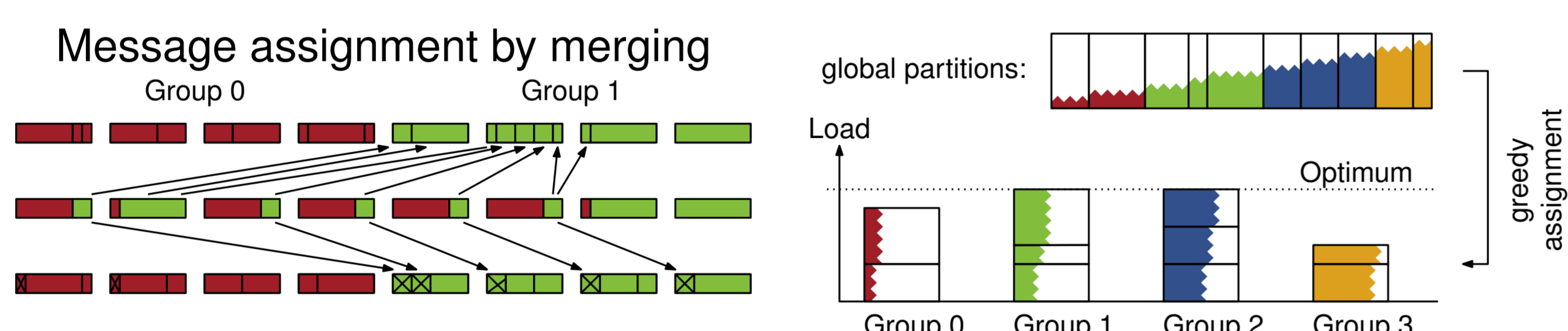
- Load balancing with space-filling curves builds down to sorting on the curve
- Sorting brings "similar" data together
- Used to build index data structures



## Requirements

- Scale to largest available machines
- Performance guarantees with asymptotic analysis
- Robustness with low overhead
- Input size
- Duplicates keys
- Distribution of input elements

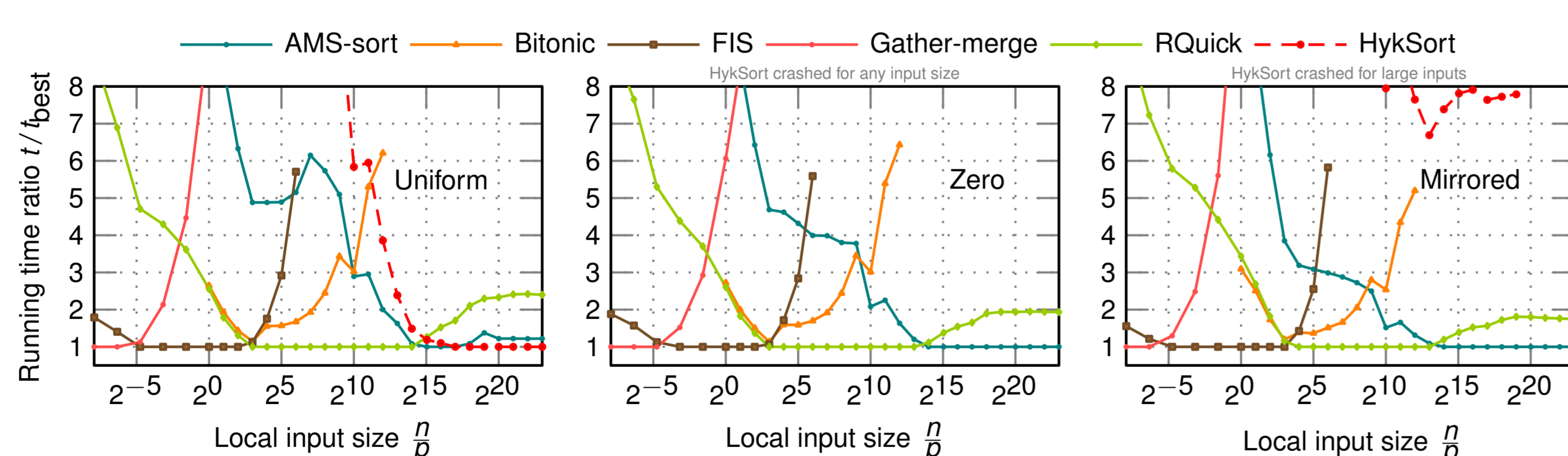
## Ideas



## Asymptotic Analysis

Algorithm	# Messages	Comm. Vol.	Remarks
Gather-merge	$\log p$	$n$	
FIS [1,2] <b>NEW</b>	$\log p$	$n/\sqrt{p}$	robust
Bitonic	$\log^2 p$	$\frac{n}{p} \log^2 p$	
HC quicksort	$\log^2 p$	$(p+1) \frac{n}{p} \log p$	best case
RQuick [2] <b>NEW</b>	$\log^2 p$	$\frac{n}{p} \log p$	robust, $p = 2^k$
JanusSort [3] <b>NEW</b>	$\log^2 p$	$\frac{n}{p} \log p$	robust
HykSort	$\geq k \log_k p$	$\geq \frac{n}{p} \log_k p$	not robust
AMS-sort [1,2] <b>NEW</b>	$\leq k \log_k p$	$\leq \frac{n}{p} \log_k p$	robust
Sample sort	$\geq p$	$\geq n/p$	+sampling cost

## Experimental Results



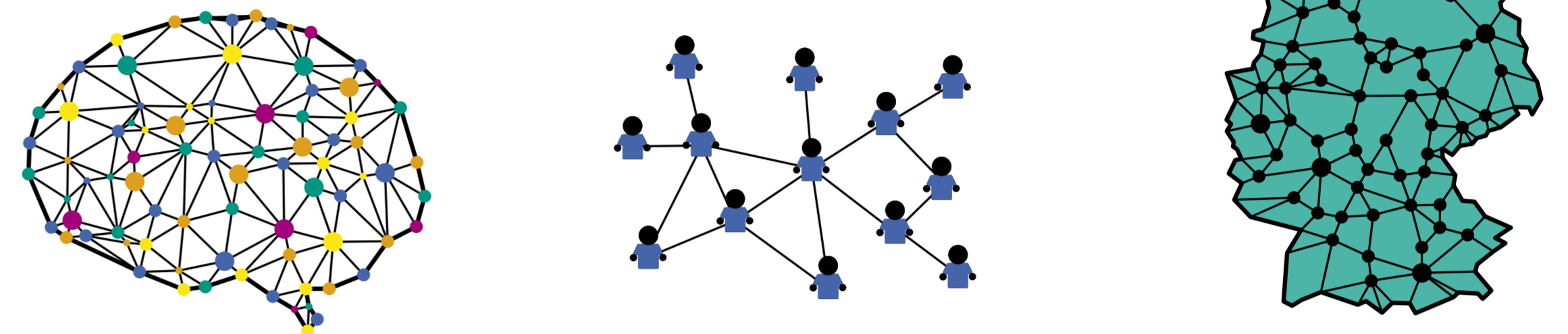
Running times of different algorithms on 262 144 cores

## References

- Wiebigke, A. and Axtmann, M., 2018. Lightweight MPI Communicators with Applications to Perfectly Balanced Quicksort. To appear at IPDPS 2018.
- Axtmann, M. and Sanders, P., 2017. Robust massively parallel sorting. ALENEX'17.
- Axtmann, M., Bingmann, T., Sanders, P., and Schulz, C., 2015. Practical massively parallel sorting. SPAA'15.

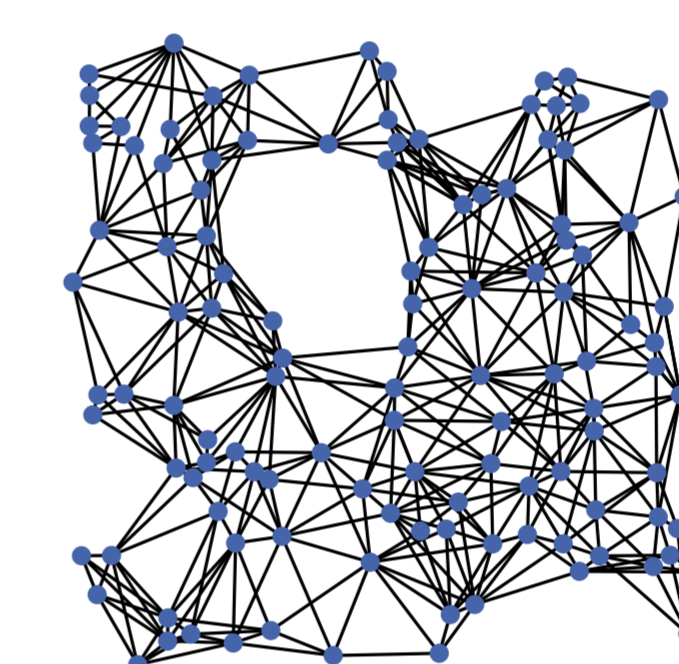
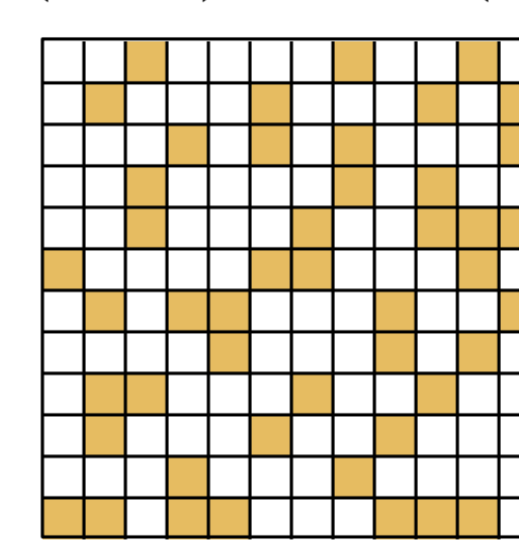
## Graph Generation

- Complex networks composed of billions of entities
- Need for algorithms capable of processing massive amounts of data
- Real-world datasets are often scarce or too small
- Graph generators provide scalable synthetic instances

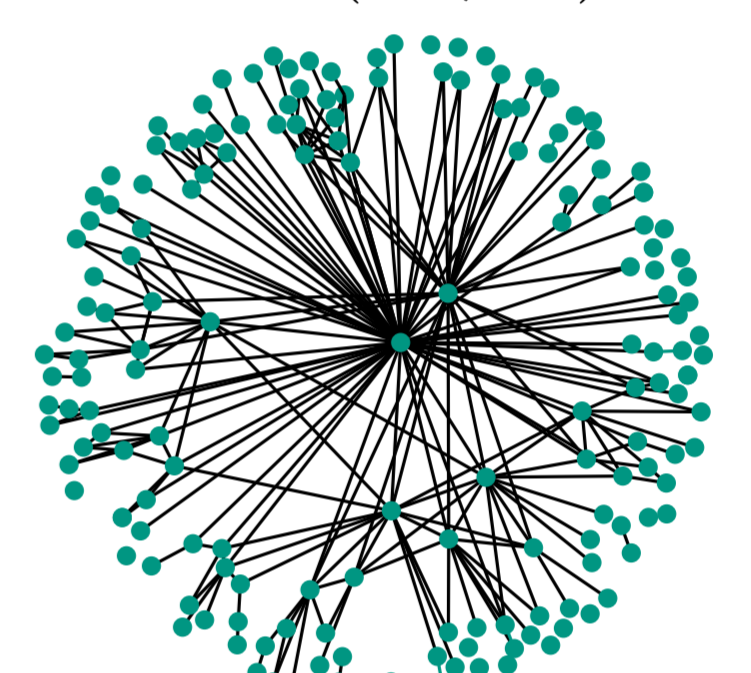


## Graph Models

Erdos-Renyi Graphs  
 $G(n, m)$  and  $G(n, p)$



Random Hyperbolic Graphs  
 $RHG(n, \gamma, \bar{d})$

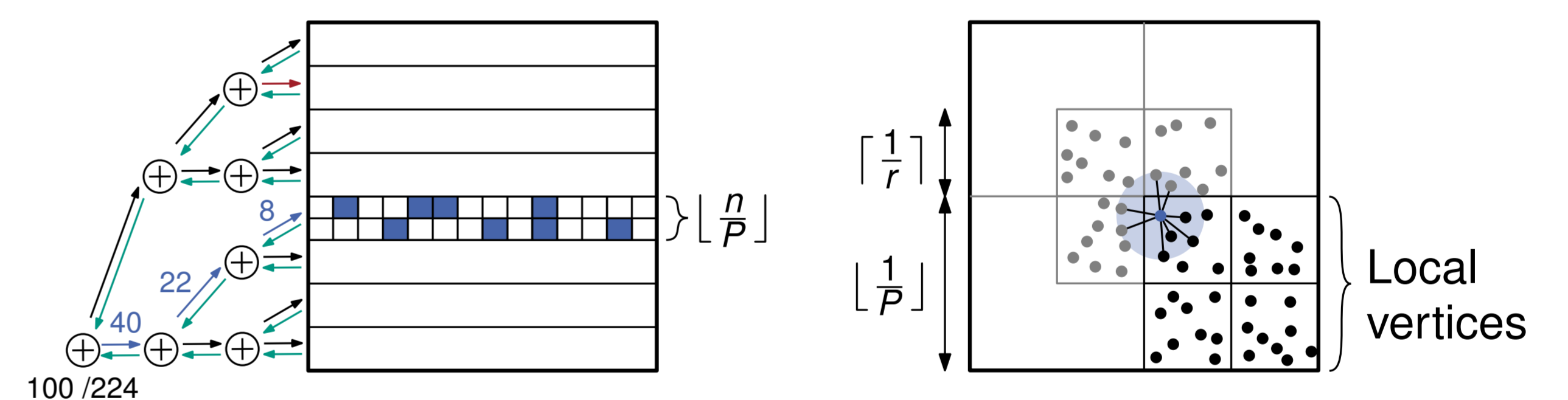


Random Geometric Graphs  $RGG(n, r)$

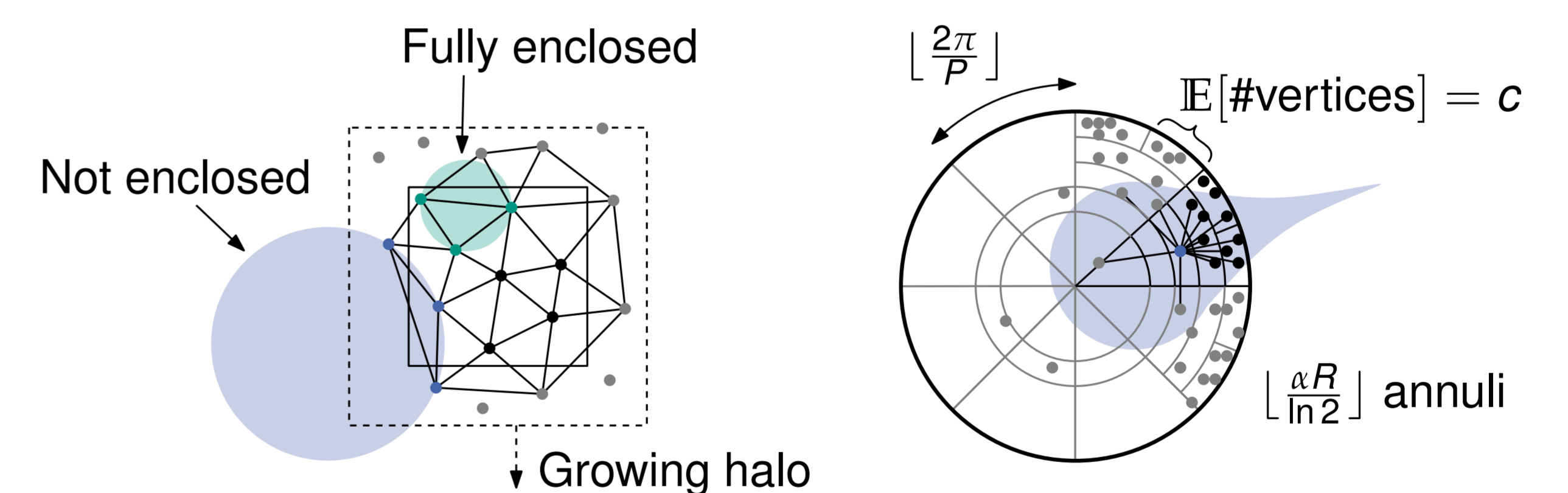
Random Delaunay Graphs  $RDG(n)$

## Zero Communication Generators

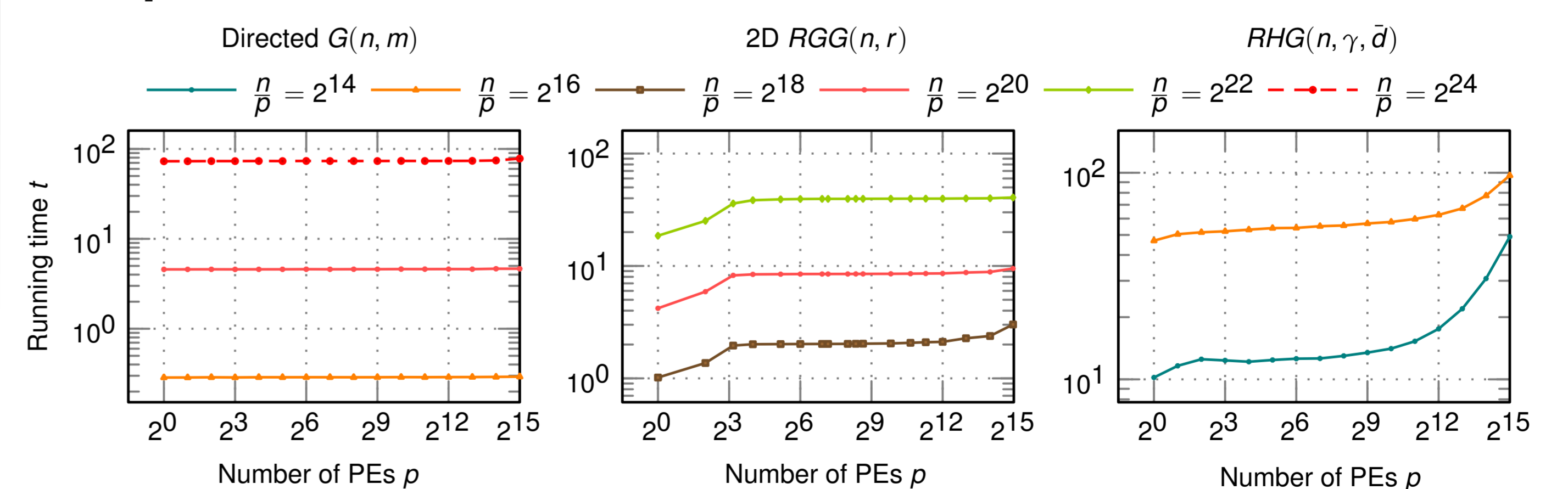
- Communication-free sampling algorithms



- Neighborhood queries using efficient recomputations



## Experimental Results



Graphs of up to  $2^{42}$  vertices and  $2^{46}$  edges in less than 20 minutes

## References

- Funke, D., Lamm, S., Sanders, P., Schulz, C., Strash, D. and von Looz, M., 2017. Communication-free massively distributed graph generation. To appear at IPDPS 2018.
- Sanders, P., Lamm, S., Hübschle-Schneider, L., Schrade, E. and Dachsbacher, C., 2017. Efficient random sampling-parallel, vectorized. In: Transactions on Mathematical Software
- Lamm, S., Sanders, P., Schulz, C. and Strash, D., 2017. Communication efficient algorithms for generating massive networks (Master thesis, Karlsruher Institut für Technologie (KIT)).